Exercise 6.1

Question 1:

Fill in the blanks using correct word given in the brackets:−

(i) All circles are __________. (congruent, similar)

(ii) All squares are __________. (similar, congruent)

(iii) All __________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are __________ and (b) their corresponding sides are __________. (equal, proportional)

Answer:

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

Question 2:

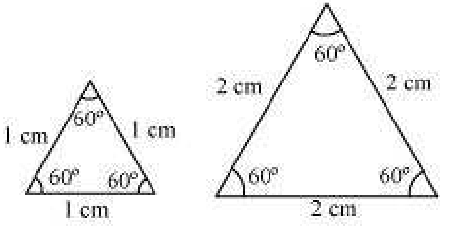
Give two different examples of pair of

(i) Similar figures

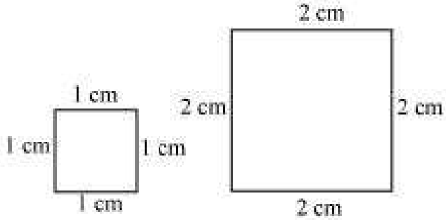
(ii)Non-similar figures

Answer:

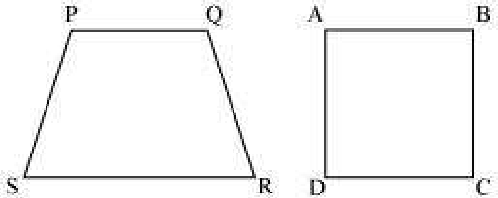
(i) Two equilateral triangles with sides 1 cm and 2 cm



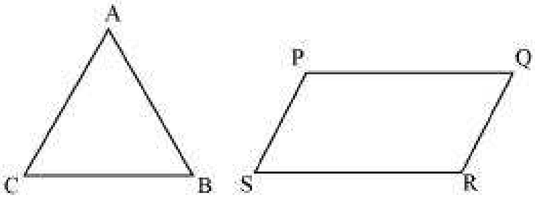
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

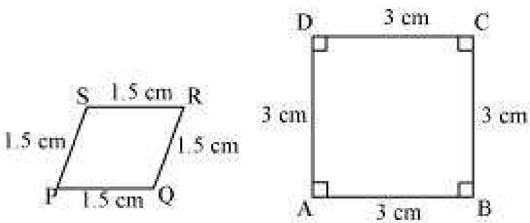


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer:

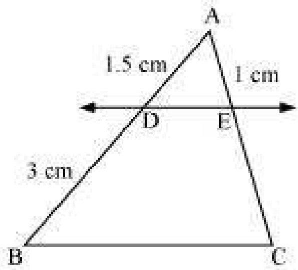
Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Exercise 6.2

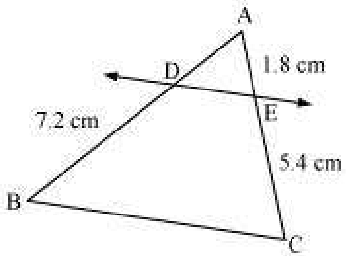
Question 1:

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i)

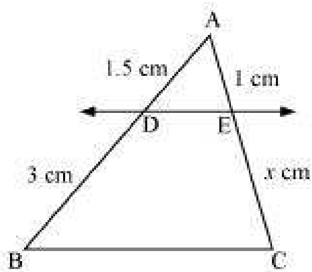


(ii)



Answer:

(i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

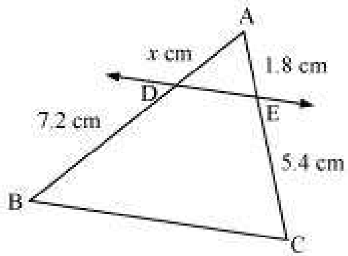
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore EC = 2 \text{ cm}$$

(ii)



Let AD = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore AD = 2.4 \text{ cm}$$

Question 2:

E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

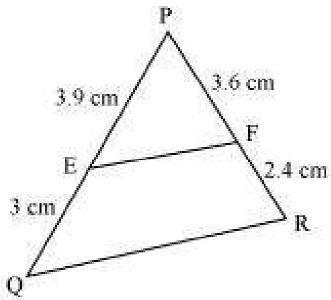
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer:

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

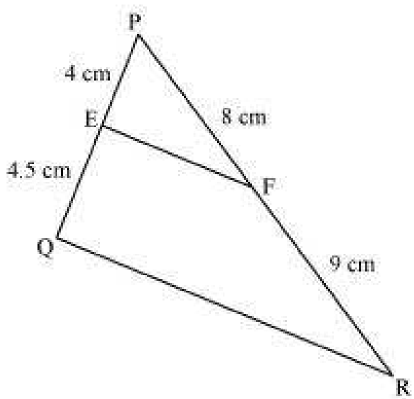
$$\frac{PE}{EQ}=\frac{3.9}{3}=1.3$$

$$\frac{PF}{FR}=\frac{3.6}{2.4}=1.5$$

Hence, $\frac{PE}{EQ}\neq\frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

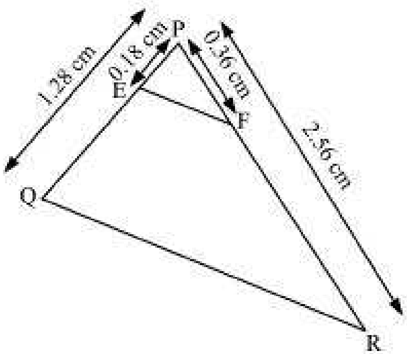
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

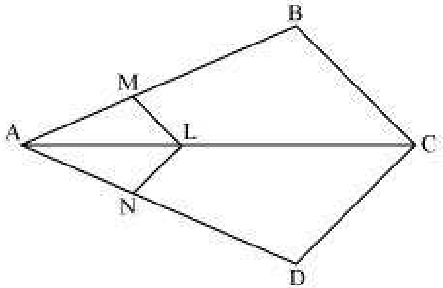
Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

Therefore, EF is parallel to QR.

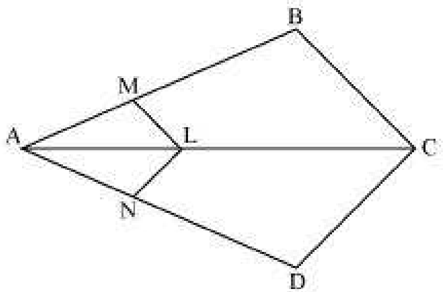
Question 3:

In the following figure, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}.$$



Answer:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \qquad (i)$$

Similarly, LN || CD

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \qquad (ii)$$

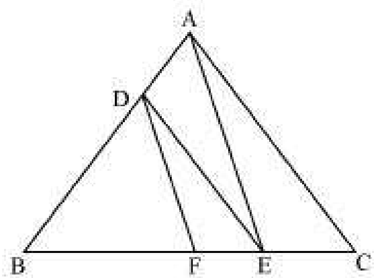
From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

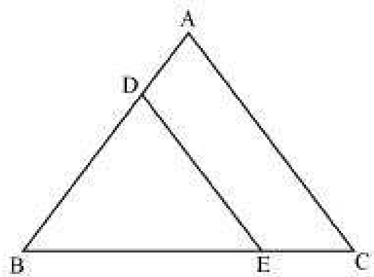
Question 4:

In the following figure, DE || AC and DF || AE. Prove that

$\dfrac{BF}{FE} = \dfrac{BE}{EC}$.

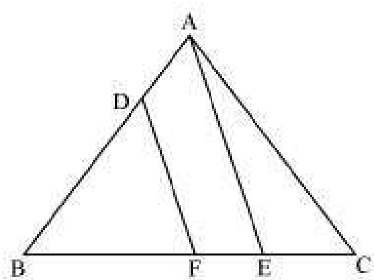


Answer:



In ΔABC, DE || AC

$\therefore \dfrac{BD}{DA} = \dfrac{BE}{EC}$ (Basic Proportionality Theorem) (i)



In ΔBAE, $DF \parallel AE$

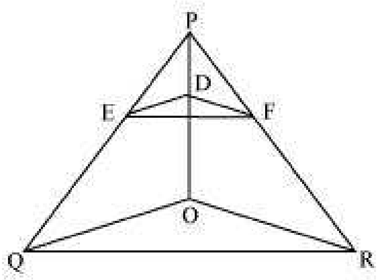
$\therefore \dfrac{BD}{DA} = \dfrac{BF}{FE}$ (Basic Proportionality Theorem) (ii)

From (i) and (ii), we obtain

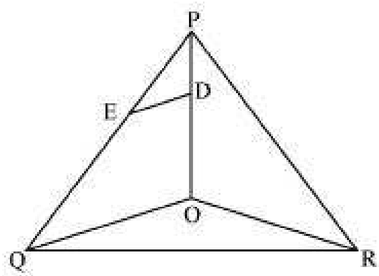
$\dfrac{BE}{EC} = \dfrac{BF}{FE}$

Question 5:

In the following figure, DE || OQ and DF || OR, show that EF || QR.

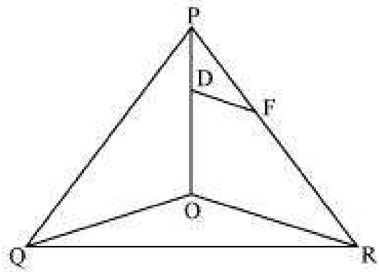


Answer:



In Δ POQ, DE || OQ

$\therefore \dfrac{PE}{EQ} = \dfrac{PD}{DO}$ (Basic proportionality theorem) (i)



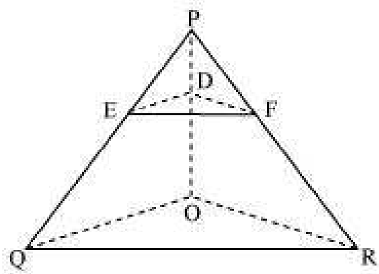
In ΔPOR, $DF \parallel OR$

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we obtain

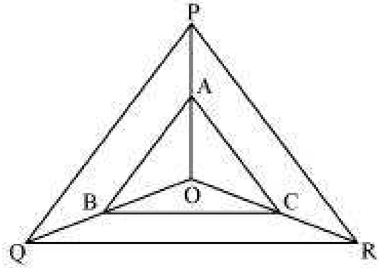
$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

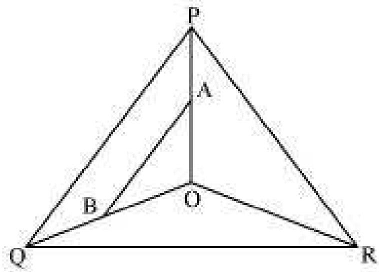


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

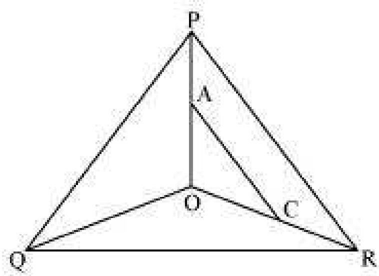


Answer:



In Δ POQ, AB || PQ

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



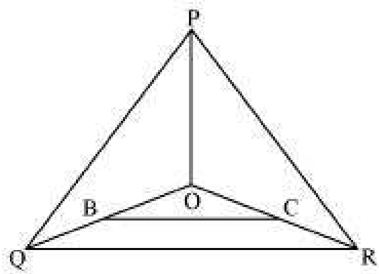
In ΔPOR, $AC \parallel PR$

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ (By basic proportionality theorem) (ii)

From (i) and (ii), we obtain

$\frac{OB}{BQ} = \frac{OC}{CR}$

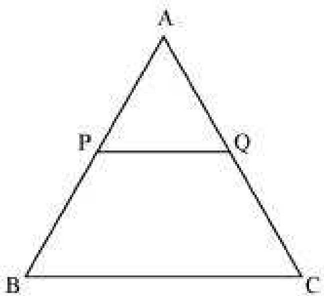
$\therefore BC \parallel QR$ (By the converse of basic proportionality theorem)



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad (\text{P is the mid-point of AB.} \therefore \text{ AP} = \text{PB})$$

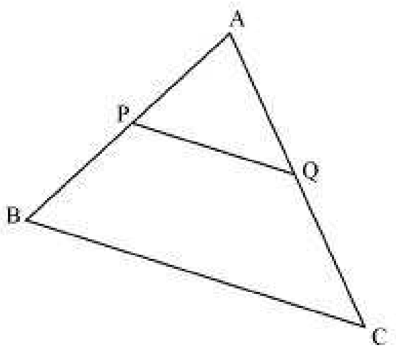
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC

It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

and $\frac{AQ}{QC} = \frac{1}{1}$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

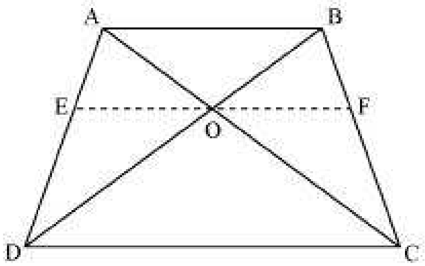
Hence, by using basic proportionality theorem, we obtain

$PQ \parallel BC$

Question 9:

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer:



Draw a line EF through point O, such that $EF \parallel CD$

In ΔADC, $EO \parallel CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$$

In ΔABD, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

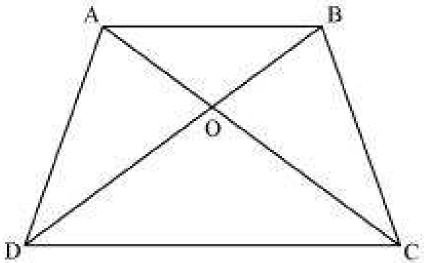
$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

Question 10:

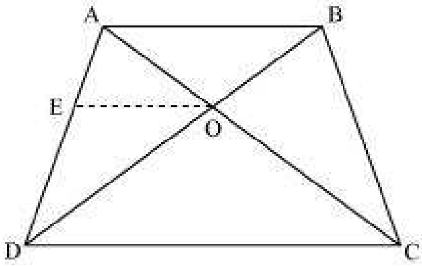
The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer:

Let us consider the following figure for the given question.



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

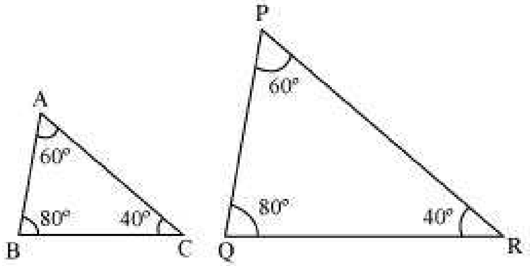
∴ ABCD is a trapezium.

Exercise 6.3

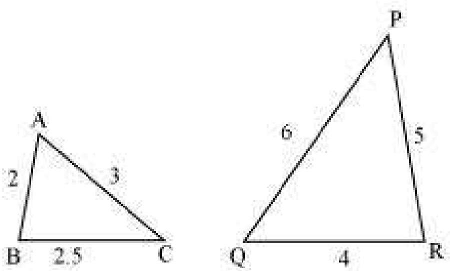
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

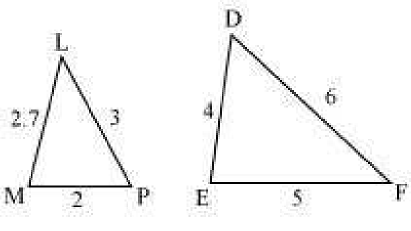
(i)



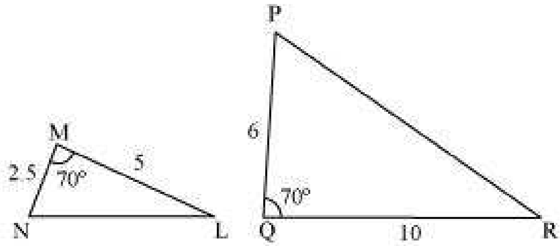
(ii)



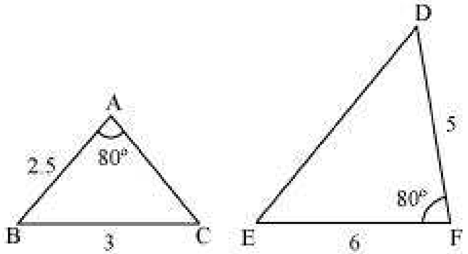
(iii)



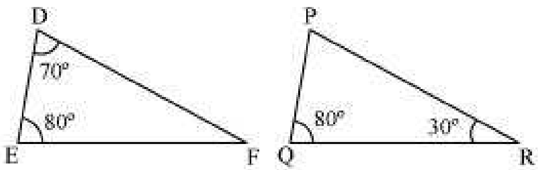
(iv)



(v)



(vi)



Answer:

(i) $\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Therefore, $\Delta ABC \sim \Delta PQR$ [By AAA similarity criterion]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii)

$\therefore\ \Delta ABC \sim \Delta QRP \qquad$ [By SSS similarity criterion]

(iii)The given triangles are not similar as the corresponding sides are not proportional.

(iv)The given triangles are not similar as the corresponding sides are not proportional.

(v)The given triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF,

$\angle D + \angle E + \angle F = 180^\circ$

(Sum of the measures of the angles of a triangle is 180º.)

$70^\circ + 80^\circ + \angle F = 180^\circ$

$\angle F = 30^\circ$

Similarly, in ΔPQR,

$\angle P + \angle Q + \angle R = 180^\circ$

(Sum of the measures of the angles of a triangle is 180º.)

$\angle P + 80^\circ + 30^\circ = 180^\circ$

$\angle P = 70^\circ$

In ΔDEF and ΔPQR,

$\angle D = \angle P$ (Each 70°)

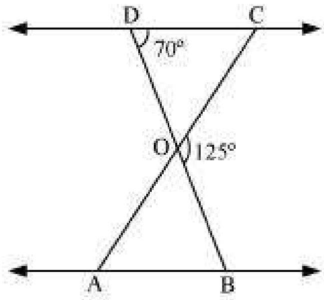
$\angle E = \angle Q$ (Each 80°)

$\angle F = \angle R$ (Each 30°)

∴ ΔDEF ~ ΔPQR [By AAA similarity criterion]

Question 2:

In the following figure, ΔODC ~ ΔOBA, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer:

DOB is a straight line.

$\therefore \angle DOC + \angle COB = 180°$

$\Rightarrow \angle DOC = 180° - 125°$

$= 55°$

In ΔDOC,

$\angle DCO + \angle CDO + \angle DOC = 180°$

(Sum of the measures of the angles of a triangle is 180º.)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO = 55°$

It is given that $\Delta ODC \sim \Delta OBA$.

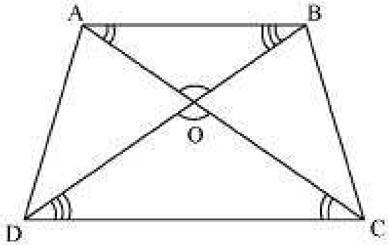
$\therefore \angle OAB = \angle OCD$ [Corresponding angles are equal in similar triangles.]

$\Rightarrow \angle OAB = 55°$

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer:



In ΔDOC and ΔBOA,

∠CDO = ∠ABO [Alternate interior angles as AB || CD]

∠DCO = ∠BAO [Alternate interior angles as AB || CD]

∠DOC = ∠BOA [Vertically opposite angles]

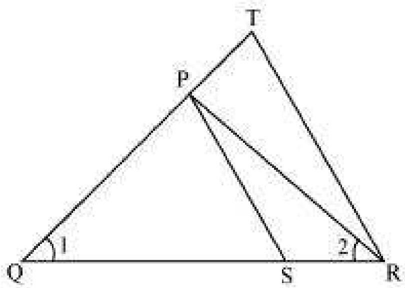
∴ ΔDOC ~ ΔBOA [AAA similarity criterion]

$\therefore \frac{DO}{BO} = \frac{OC}{OA}$ [Corresponding sides are proportional]

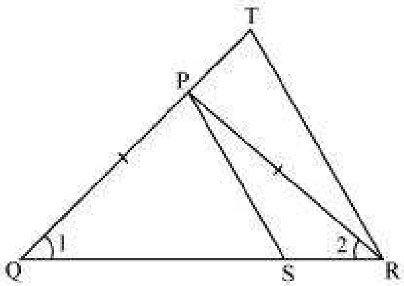
$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2.$ Show that $\Delta PQS \sim \Delta TQR$



Answer:



In ΔPQR, $\angle PQR = \angle PRQ$

$\therefore PQ = PR$ (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad (ii)$$

In ΔPQS and ΔTQR,

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad \left[\text{Using}(ii)\right]$$

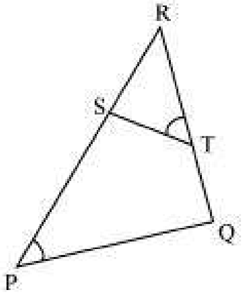
$$\angle Q = \angle Q$$

$$\therefore \ \Delta PQS \sim \Delta TQR \qquad [\text{SAS similarity criterion}]$$

Question 5:

S and T are point on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Answer:



In ΔRPQ and ΔRST,

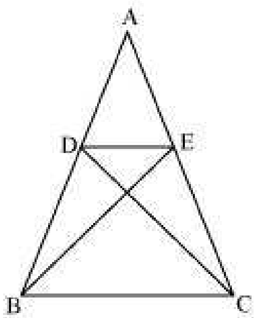
∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

Question 6:

In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Answer:

It is given that ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] (1)

And, AD = AE [By CPCT] (2)

In ΔADE and ΔABC,

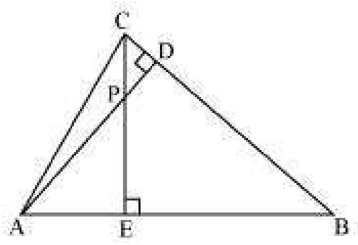
$\frac{AD}{AB} = \frac{AE}{AC}$ [Dividing equation (2) by (1)]

∠A = ∠A [Common angle]

∴ ΔADE ~ ΔABC [By SAS similarity criterion]

Question 7:

In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) ΔAEP ~ ΔCDP

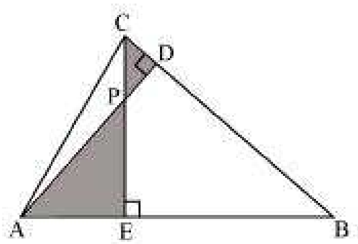
(ii) ΔABD ~ ΔCBE

(iii) ΔAEP ~ ΔADB

(v) ΔPDC ~ ΔBEC

Answer:

(i)



In ΔAEP and ΔCDP,

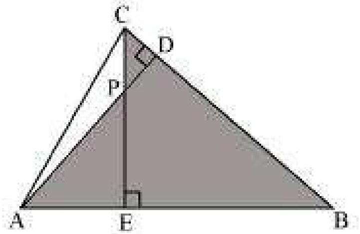
∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔCDP

(ii)



In ΔABD and ΔCBE,

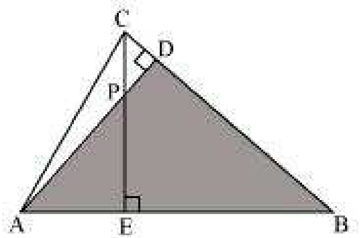
∠ADB = ∠CEB (Each 90°)

∠ABD = ∠CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ~ ΔCBE

(iii)



In ΔAEP and ΔADB,

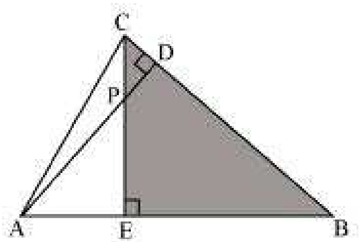
∠AEP = ∠ADB (Each 90°)

∠PAE = ∠DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔADB

(iv)



In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ (Each 90°)

$\angle PCD = \angle BCE$ (Common angle)

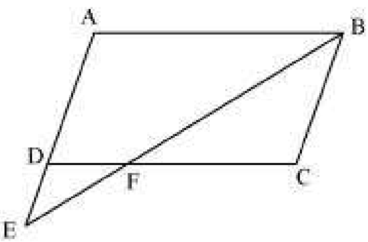
Hence, by using AA similarity criterion,

$\Delta PDC \sim \Delta BEC$

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$

Answer:



In ΔABE and ΔCFB,

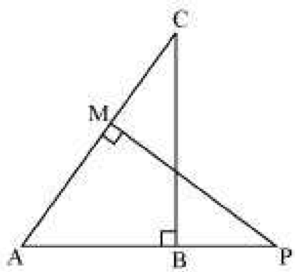
$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles as AE || BC)

$\therefore$ $\Delta ABE \sim \Delta CFB$ (By AA similarity criterion)

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) $\Delta ABC \sim \Delta AMP$

(ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$

Answer:

In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ ΔABC ~ ΔAMP (By AA similarity criterion)

$\Rightarrow \dfrac{CA}{PA} = \dfrac{BC}{MP}$ (Corresponding sides of similar triangles are proportional)

Question 10:

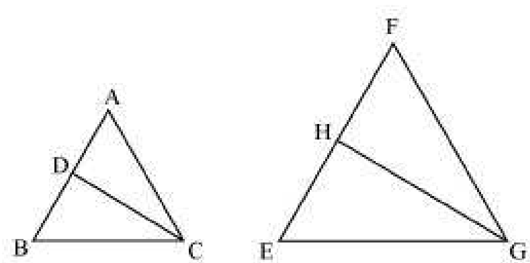
CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$

(ii) ΔDCB ~ ΔHGE

(iii) ΔDCA ~ ΔHGF

Answer:



It is given that ΔABC ~ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,

$\angle A = \angle F$ (Proved above)

$\angle ACD = \angle FGH$ (Proved above)

$\therefore \Delta ACD \sim \Delta FGH$ (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In ΔDCB and ΔHGE,

$\angle DCB = \angle HGE$ (Proved above)

$\angle B = \angle E$ (Proved above)

$\therefore \Delta DCB \sim \Delta HGE$ (By AA similarity criterion)

In ΔDCA and ΔHGF,

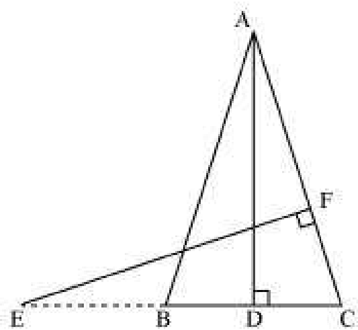
$\angle ACD = \angle FGH$ (Proved above)

$\angle A = \angle F$ (Proved above)

$\therefore \Delta DCA \sim \Delta HGF$ (By AA similarity criterion)

Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF $\perp$ AC, prove that $\Delta ABD \sim \Delta ECF$



Answer:

It is given that ABC is an isosceles triangle.

$\therefore AB = AC$

$\Rightarrow \angle ABD = \angle ECF$

In ΔABD and ΔECF,

∠ADB = ∠EFC (Each 90°)

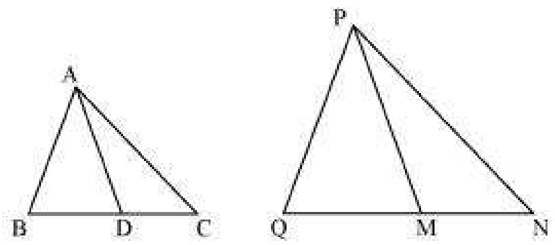
∠BAD = ∠CEF (Proved above)

∴ ΔABD ~ ΔECF (By using AA similarity criterion)

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that ΔABC ~ ΔPQR.

Answer:



Median divides the opposite side.

∴ $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (Proved above)

∴ ΔABD ~ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

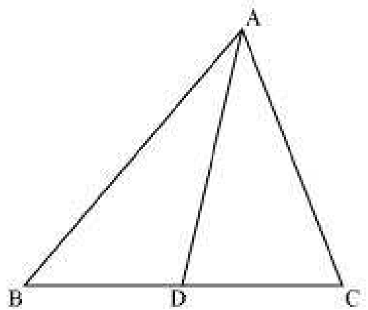
$\frac{AB}{PQ} = \frac{BC}{QR}$

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that $CA^2 = CB.CD.$

Answer:



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Given)

∠ACD = ∠BCA (Common angle)

∴ ΔADC ~ ΔBAC (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

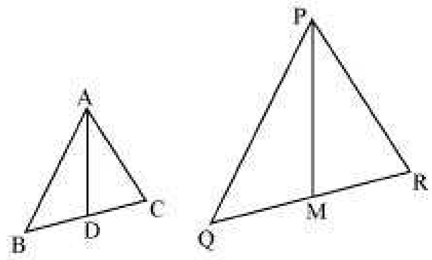
$\therefore \frac{CA}{CB} = \frac{CD}{CA}$

$\Rightarrow CA^2 = CB \times CD$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

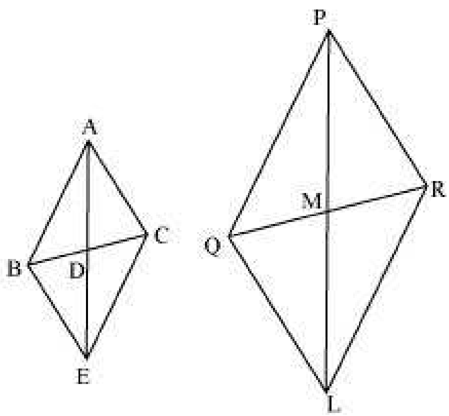
Answer:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

∴ $\Delta ABE \sim \Delta PQL$ (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ $\angle BAE = \angle QPL$ … (1)

Similarly, it can be proved that $\Delta AEC \sim \Delta PLR$ and

$\angle CAE = \angle RPL$ … (2)

Adding equation (1) and (2), we obtain

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\Rightarrow \angle CAB = \angle RPQ$ … (3)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

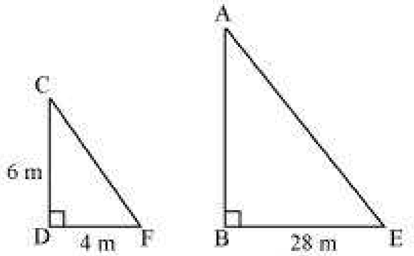
$\angle CAB = \angle RPQ$ [Using equation (3)]

∴ $\Delta ABC \sim \Delta PQR$ (By SAS similarity criterion)

Question 15:

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

$\therefore \Delta ABE \sim \Delta CDF$ (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6\text{ m}} = \frac{28}{4}$$

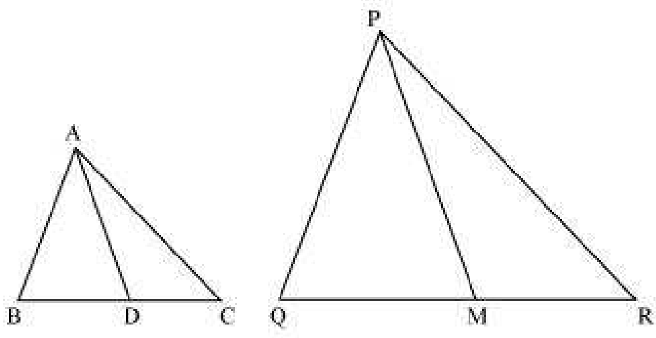
$$\Rightarrow AB = 42\text{ m}$$

Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ prove tha t $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer:



It is given that ΔABC ~ ΔPQR

We know that the corresponding sides of similar triangles are in proportion.

∴ $\frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}$ … (1)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PM are medians, they will divide their opposite sides.

∴ $BD=\frac{BC}{2}$ and $QM=\frac{QR}{2}$ … (3)

From equations (1) and (3), we obtain

$\frac{AB}{PQ}=\frac{BD}{QM}$ … (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

$\frac{AB}{PQ}=\frac{BD}{QM}$ [Using equation (4)]

∴ ΔABD ~ ΔPQM (By SAS similarity criterion)

⇒ $\frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$

Exercise 6.4

Question 1:

Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Answer:

It is given that $\Delta ABC \sim \Delta DEF$.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$EF = 15.4$ cm,

$ar(\Delta ABC) = 64\ cm^2$,

$ar(\Delta DEF) = 121\ cm^2$

$$\therefore \frac{ar(ABC)}{ar(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64\ cm^2}{121\ cm^2}\right) = \frac{BC^2}{(15.4\ cm)^2}$$

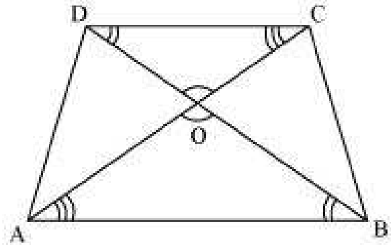
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right) cm$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right) cm = (8 \times 1.4) cm = 11.2\ cm$$

Question 2:

Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Answer:



Since AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

∴ ΔAOB ~ ΔCOD (By AAA similarity criterion)

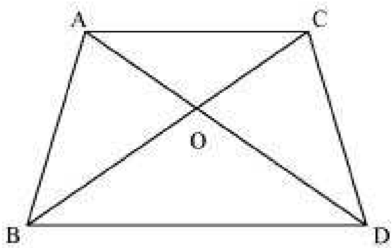
$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$$

Since AB = 2 CD,

$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{2\ CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

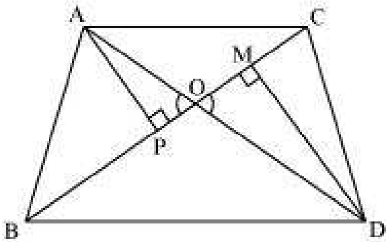
Question 3:

In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AO}{DO}$



Answer:

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2}\times\text{Base}\times\text{Height}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{\frac{1}{2}BC\times AP}{\frac{1}{2}BC\times DM}=\frac{AP}{DM}$$.

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO ~ ΔDMO (By AA similarity criterion)

$$\therefore \frac{AP}{DM}=\frac{AO}{DO}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{DO}$$

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.

Answer:

Let us assume two similar triangles as ΔABC ~ ΔPQR.

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{AC}{PR}\right)^2 \qquad (1)$$

Given that, ar $(\Delta ABC) =$ ar (ΔPQR)

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)}=1$$

Putting this value in equation (1), we obtain

$$1=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{AC}{PR}\right)^2$$

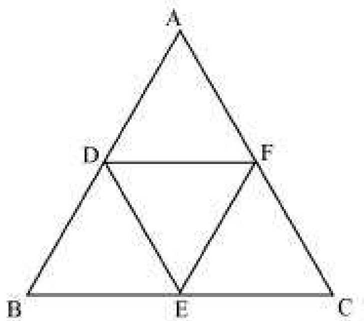
$\Rightarrow AB = PQ, BC = QR,$ and $AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ $\qquad$ (By SSS congruence criterion)

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Answer:



D and E are the mid-points of ΔABC.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2}AC$

In ΔBED and ΔBCA,

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

$\therefore \Delta BED \sim \Delta BCA$ (AAA similarity criterion)

$$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BED) = \frac{1}{4} ar(\Delta BCA)$$

Similarly, $ar(\Delta CFE) = \frac{1}{4} ar(CBA)$ and $ar(\Delta ADF) = \frac{1}{4} ar(\Delta ABC)$

Also, $ar(\Delta DEF) = ar(\Delta ABC) - [ar(\Delta BED) + ar(\Delta CFE) + ar(\Delta ADF)]$

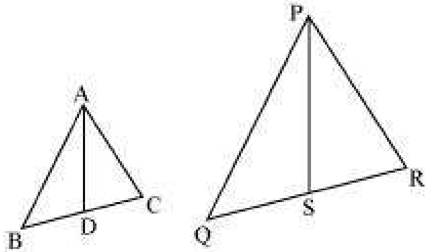
$$\Rightarrow ar(\Delta DEF) = ar(\Delta ABC) - \frac{3}{4} ar(\Delta ABC) = \frac{1}{4} ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer:



Let us assume two similar triangles as ΔABC ~ ΔPQR. Let AD and PS be the medians of these triangles.

∵ ΔABC ~ ΔPQR

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ...(1)

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ... (2)

Since AD and PS are medians,

∴ BD = DC = $\frac{BC}{2}$

And, QS = SR = $\frac{QR}{2}$

Equation (1) becomes

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ ... (3)

In ΔABD and ΔPQS,

∠B = ∠Q [Using equation (2)]

And, $\frac{AB}{PQ} = \frac{BD}{QS}$ [Using equation (3)]

∴ ΔABD ~ ΔPQS (SAS similarity criterion)

Therefore, it can be said that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$ ... (4)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

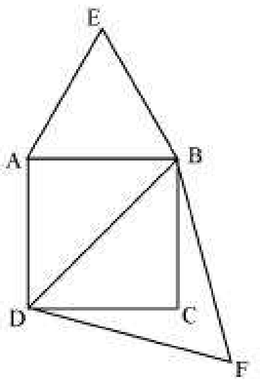
And hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer:



Let ABCD be a square of side a.

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF.

Side of an equilateral triangle, ΔABE, described on one of its sides = *a*

Side of an equilateral triangle, ΔDBF, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \Delta\,\text{ABE}}{\text{Area of } \Delta\,\text{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Question 8:

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

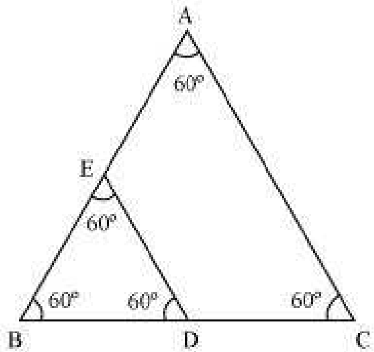
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Answer:



We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of ΔABC = x

Therefore, side of $\Delta\text{BDE} = \frac{x}{2}$

$$\therefore \quad \frac{\text{area}(\Delta \text{ABC})}{\text{area}(\Delta \text{BDE})} = \left(\frac{x}{\frac{x}{2}} \right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Question 9:

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Answer:

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9} \right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Exercise 6.5

Question 1:

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer:

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

49 + 576 = 625

Or, $7^2 + 24^2 = 25^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

However, $9 + 36 \neq 64$

Or, $3^2 + 6^2 \neq 8^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii)Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

However, $2500 + 6400 \neq 10000$

Or, $50^2 + 80^2 \neq 100^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv)Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, 144 +25 = 169

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

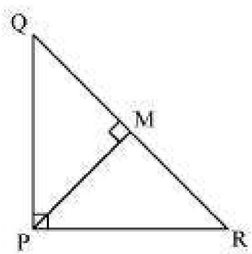
We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Question 2:

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2 = QM \times MR$.

Answer:



Let $\angle MPR = x$

In ΔMPR,

$\angle MRP = 180^\circ - 90^\circ - x$

$\angle MRP = 90^\circ - x$

Similarly, in ΔMPQ,

$\angle MPQ = 90^\circ - \angle MPR$

$= 90^\circ - x$

$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$

$\angle MQP = x$

In Δ QMP and ΔPMR,

$\angle MPQ = \angle MRP$

$\angle PMQ = \angle RMP$

$\angle MQP = \angle MPR$

$\therefore \Delta QMP \sim \Delta PMR$ (By AAA similarity criterion)

$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$

$\Rightarrow PM^2 = QM \times MR$

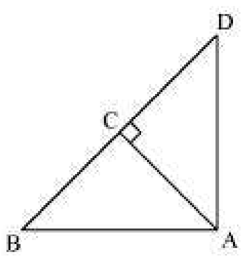
Question 3:

In the following figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC \times BD$

(ii) $AC^2 = BC \times DC$

(iii) $AD^2 = BD \times CD$



Answer:

(i) In ΔADB and ΔCAB,

$\angle DAB = \angle ACB \qquad (\text{Each } 90^\circ)$

$\angle ABD = \angle CBA \qquad (\text{Common angle})$

$\therefore\ \Delta ADB \sim \Delta CAB$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{CB} = \frac{BD}{AB}$$

$\Rightarrow AB^2 = CB \times BD$

(ii) Let $\angle CAB = x$

In ΔCBA,

$\angle CBA = 180^\circ - 90^\circ - x$

$\angle CBA = 90^\circ - x$

Similarly, in ΔCAD,

$\angle CAD = 90^\circ - \angle CAB$

$\qquad = 90^\circ - x$

$\angle CDA = 180^\circ - 90^\circ - (90^\circ - x)$

$\angle CDA = x$

In ΔCBA and ΔCAD,

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA \qquad (\text{Each } 90^\circ)$

$\therefore \Delta CBA \sim \Delta CAD \qquad (\text{By AAA rule})$

$$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$$

$\Rightarrow AC^2 = DC \times BC$

(iii) In ΔDCA and ΔDAB,

∠DCA = ∠ DAB (Each 90º)

∠CDA = ∠ ADB (Common angle)

$\therefore \Delta DCA \sim \Delta DAB$ (AA similarity criterion)

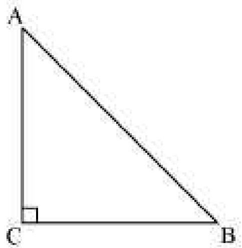
$$\Rightarrow \frac{DC}{DA} = \frac{DA}{DB}$$

$$\Rightarrow AD^2 = BD \times CD$$

Question 4:

ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2\ AC^2$.

Answer:



Given that ΔABC is an isosceles triangle.

∴ AC = CB

Applying Pythagoras theorem in ΔABC (i.e., right-angled at point C), we obtain

$$AC^2 + CB^2 = AB^2$$

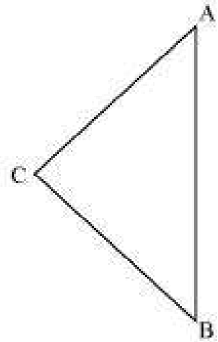
$$\Rightarrow AC^2 + AC^2 = AB^2 \quad (AC = CB)$$

$$\Rightarrow 2AC^2 = AB^2$$

Question 5:

ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

Answer:



Given that,

$$AB^2 = 2AC^2$$

$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2 \quad (\text{As } AC = BC)$$

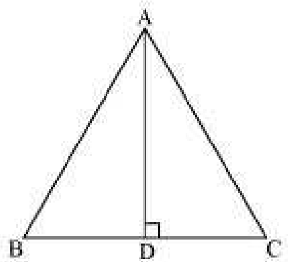
The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Question 6:

ABC is an equilateral triangle of side 2*a*. Find each of its altitudes.

Answer:



Let AD be the altitude in the given equilateral triangle, ΔABC.

We know that altitude bisects the opposite side.

∴ BD = DC = *a*

In ΔADB,

$\angle ADB = 90°$

Applying pythagoras theorem, we obtain

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 + a^2 = (2a)^2$

$\Rightarrow AD^2 + a^2 = 4a^2$

$\Rightarrow AD^2 = 3a^2$

$\Rightarrow AD = a\sqrt{3}$

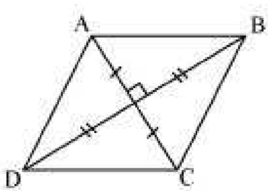
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Question 7:

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer:



In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$$AB^2 = AO^2 + OB^2 \quad \dots (1)$$

$$BC^2 = BO^2 + OC^2 \quad \dots (2)$$

$$CD^2 = CO^2 + OD^2 \quad \dots (3)$$

$$AD^2 = AO^2 + OD^2 \quad \dots (4)$$

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

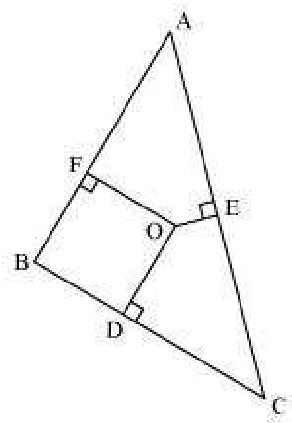
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

Question 8:

In the following figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

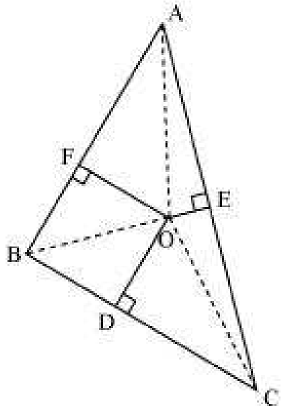


(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer:

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in ΔAOF, we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in ΔBOD,

$$OB^2 = OD^2 + BD^2$$

Similarly, in ΔCOE,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

(ii) From the above result,

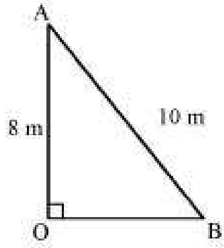
$$AF^2 + BD^2 + EC^2 = \left(OA^2 - OE^2\right) + \left(OC^2 - OD^2\right) + \left(OB^2 - OF^2\right)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer:



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$$AB^2 = OA^2 + BO^2$$

$$(10\text{ m})^2 = (8\text{ m})^2 + OB^2$$

$$100\text{ m}^2 = 64\text{ m}^2 + OB^2$$

$$OB^2 = 36\text{ m}^2$$

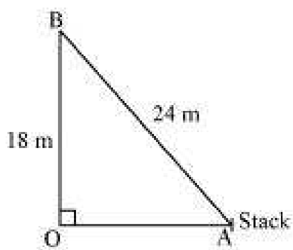
$$OB = 6\text{ m}$$

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer:



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

$$AB^2 = OB^2 + OA^2$$

$$(24\text{ m})^2 = (18\text{ m})^2 + OA^2$$

$$OA^2 = (576 - 324)\text{m}^2 = 252\text{ m}^2$$

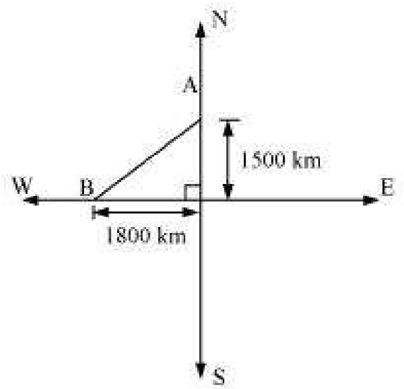
$$OA = \sqrt{252}\text{ m} = \sqrt{6\times6\times7}\text{ m} = 6\sqrt{7}\text{ m}$$

Therefore, the distance from the base is $6\sqrt{7}$ m.

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer:



Distance travelled by the plane flying towards north in $1\frac{1}{2}\text{hrs}$

$$= 1{,}000 \times 1\frac{1}{2} = 1{,}500\text{ km}$$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}\text{hrs}$

$$= 1{,}200 \times 1\frac{1}{2} = 1{,}800\text{ km}$$

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

Distance between these planes after $1\frac{1}{2}$ hrs, AB = $\sqrt{OA^2 + OB^2}$

$$= \left(\sqrt{(1,500)^2 + (1,800)^2}\right)\text{km} = \left(\sqrt{2250000 + 3240000}\right)\text{km}$$

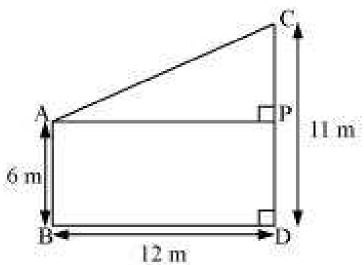
$$= \left(\sqrt{5490000}\right)\text{km} = \left(\sqrt{9 \times 610000}\right)\text{km} = 300\sqrt{61}\text{ km}$$

Therefore, the distance between these planes will be $300\sqrt{61}$ km after $1\frac{1}{2}$ hrs.

Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer:



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, CP = 11 − 6 = 5 m

From the figure, it can be observed that AP = 12m

Applying Pythagoras theorem for ΔAPC, we obtain

$$AP^2 + PC^2 = AC^2$$

$$(12\text{ m})^2 + (5\text{ m})^2 = AC^2$$

$$AC^2 = (144 + 25)\text{m}^2 = 169\text{ m}^2$$

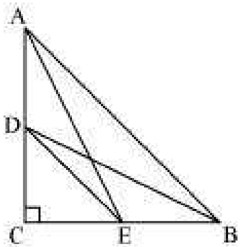
$$AC = 13\text{ m}$$

Therefore, the distance between their tops is 13 m.

Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Answer:



Applying Pythagoras theorem in ΔACE, we obtain

$$AC^2 + CE^2 = AE^2 \quad \text{... (1)}$$

Applying Pythagoras theorem in ΔBCD, we obtain

$$BC^2 + CD^2 = BD^2 \quad \text{... (2)}$$

Using equation (1) and equation (2), we obtain

$$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \text{... (3)}$$

Applying Pythagoras theorem in ΔCDE, we obtain

$$DE^2 = CD^2 + CE^2$$

Applying Pythagoras theorem in ΔABC, we obtain

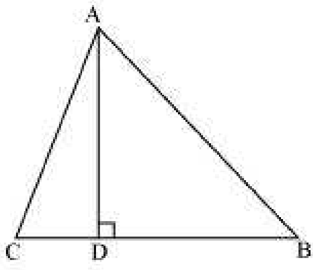
$$AB^2 = AC^2 + CB^2$$

Putting the values in equation (3), we obtain

$$DE^2 + AB^2 = AE^2 + BD^2$$

Question 14:

The perpendicular from A on side BC of a ΔABC intersect BC at D such that DB = 3 CD. Prove that 2 $AB^2 = 2\ AC^2 + BC^2$



Answer:

Applying Pythagoras theorem for ΔACD, we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \qquad \dots (1)$$

Applying Pythagoras theorem in ΔABD, we obtain

$$AB^2 = AD^2 + DB^2$$

$$AD^2 = AB^2 - DB^2 \qquad \dots (2)$$

From equation (1) and equation (2), we obtain

$$AC^2 - DC^2 = AB^2 - DB^2 \qquad \dots (3)$$

It is given that 3DC = DB

$$\therefore DC = \frac{BC}{4} \text{ and } DB = \frac{3BC}{4}$$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

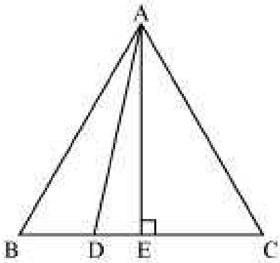
$$16AB^2 - 16AC^2 = 8BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

Question 15:

In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that 9 AD^2 = 7 AB^2.

Answer:



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

$\therefore$ BE = EC = $\frac{BC}{2} = \frac{a}{2}$

And, AE = $\frac{a\sqrt{3}}{2}$

Given that, BD = $\frac{1}{3}$ BC

$\therefore$ BD = $\frac{a}{3}$

DE = BE − BD = $\frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

Applying Pythagoras theorem in ΔADE, we obtain

$AD^2 = AE^2 + DE^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

$$= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right)$$

$$= \frac{28a^2}{36}$$

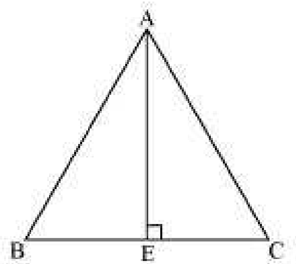
$$= \frac{7}{9}AB^2$$

$\Rightarrow 9\ AD^2 = 7\ AB^2$

Question 16:

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer:



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$

Applying Pythagoras theorem in ΔABE, we obtain

$AB^2 = AE^2 + BE^2$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$4AE^2 = 3a^2$

⇒ 4 × (Square of altitude) = 3 × (Square of one side)

Question 17:

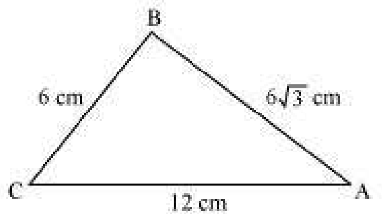
Tick the correct answer and justify: In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Answer:



Given that, AB = $6\sqrt{3}$ cm, AC = 12 cm, and BC = 6 cm

It can be observed that

$AB^2 = 108$

$AC^2 = 144$

And, $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

The given triangle, ΔABC, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.

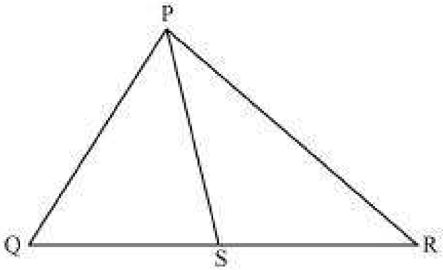
$\therefore \angle B = 90°$

Hence, the correct answer is (C).

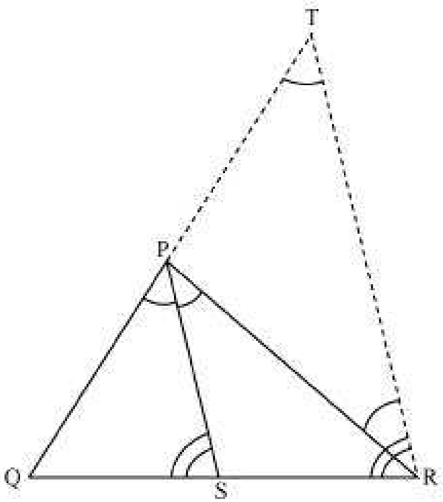
Exercise 6.6

Question 1:

In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.



Answer:



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of ∠QPR.

∠QPS = ∠SPR … (1)

By construction,

∠SPR = ∠PRT (As PS || TR) … (2)

∠QPS = ∠QTR (As PS || TR) … (3)

Using these equations, we obtain

$\angle PRT = \angle QTR$

$\therefore$ PT = PR

By construction,

PS || TR

By using basic proportionality theorem for ΔQTR,

$$\frac{QS}{SR} = \frac{QP}{PT}$$

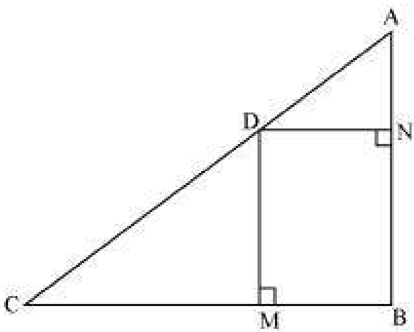
$$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR} \qquad (PT = TR)$$

Question 2:

In the given figure, D is a point on hypotenuse AC of ΔABC, DM ⊥ BC and DN ⊥ AB, Prove that:

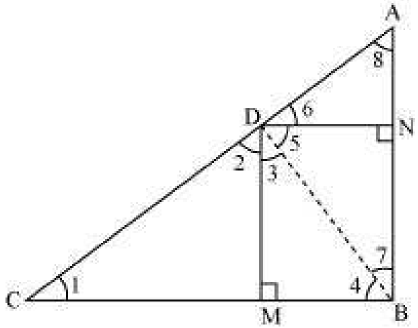
(i) $DM^2 = DN.MC$

(ii) $DN^2 = DM.AN$



Answer:

(i)Let us join DB.



We have, DN || CB, DM || AB, and $\angle B = 90°$

∴ DMBN is a rectangle.

∴ DN = MB and DM = NB

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

∴ $\angle CDB = 90°$

$\Rightarrow \angle 2 + \angle 3 = 90°$ … (1)

In ΔCDM,

$\angle 1 + \angle 2 + \angle DMC = 180°$

$\Rightarrow \angle 1 + \angle 2 = 90°$ … (2)

In ΔDMB,

$\angle 3 + \angle DMB + \angle 4 = 180°$

$\Rightarrow \angle 3 + \angle 4 = 90°$ … (3)

From equation (1) and (2), we obtain

$\angle 1 = \angle 3$

From equation (1) and (3), we obtain

$\angle 2 = \angle 4$

In ΔDCM and ΔBDM,

$\angle 1 = \angle 3$ (Proved above)

$\angle 2 = \angle 4$ (Proved above)

∴ ΔDCM ~ ΔBDM (AA similarity criterion)

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$

$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \qquad (BM = DN)$$

$\Rightarrow DM^2 = DN \times MC$

(ii) In right triangle DBN,

$\angle 5 + \angle 7 = 90° \ldots (4)$

In right triangle DAN,

$\angle 6 + \angle 8 = 90° \ldots (5)$

D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle ADB = 90°$

$\Rightarrow \angle 5 + \angle 6 = 90° \ldots (6)$

From equation (4) and (6), we obtain

$\angle 6 = \angle 7$

From equation (5) and (6), we obtain

$\angle 8 = \angle 5$

In ΔDNA and ΔBND,

$\angle 6 = \angle 7$ (Proved above)

$\angle 8 = \angle 5$ (Proved above)

$\therefore \Delta DNA \sim \Delta BND$ (AA similarity criterion)

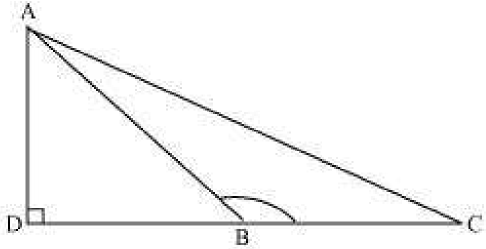
$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

$\Rightarrow DN^2 = AN \times NB$

$\Rightarrow DN^2 = AN \times DM$ (As NB = DM)

Question 3:

In the given figure, ABC is a triangle in which $\angle ABC > 90°$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer:

Applying Pythagoras theorem in ΔADB, we obtain

$AB^2 = AD^2 + DB^2$ … (1)

Applying Pythagoras theorem in ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

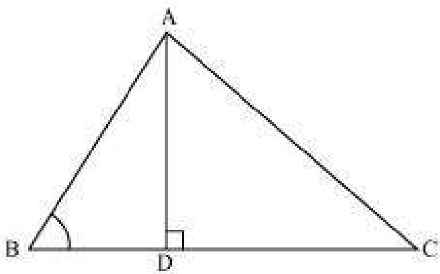
$AC^2 = AD^2 + (DB + BC)^2$

$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$AC^2 = AB^2 + BC^2 + 2DB \times BC$ [Using equation (1)]

Question 4:

In the given figure, ABC is a triangle in which $\angle ABC < 90°$ and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Answer:

Applying Pythagoras theorem in ΔADB, we obtain

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 = AB^2 - DB^2$ … (1)

Applying Pythagoras theorem in ΔADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 - BD^2 + DC^2 = AC^2$ [Using equation (1)]

$AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$

$= AB^2 + BC^2 - 2BC \times BD$

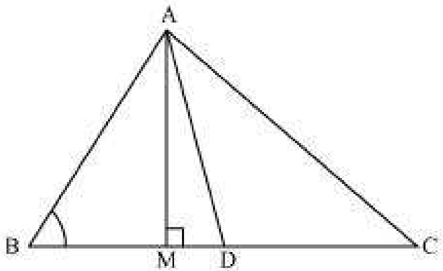
Question 5:

In the given figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that:

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Answer:

(i) Applying Pythagoras theorem in ΔAMD, we obtain

$AM^2 + MD^2 = AD^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$

$AM^2 + (MD + DC)^2 = AC^2$

$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$AD^2 + DC^2 + 2MD.DC = AC^2$ [Using equation (1)]

Using the result, $DC = \frac{BC}{2}$, we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in ΔABM, we obtain

$AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii)Applying Pythagoras theorem in ΔABM, we obtain

$AM^2 + MB^2 = AB^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$ … (2)

Adding equations (1) and (2), we obtain

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD\,(-BD + DC) = AB^2 + AC^2$

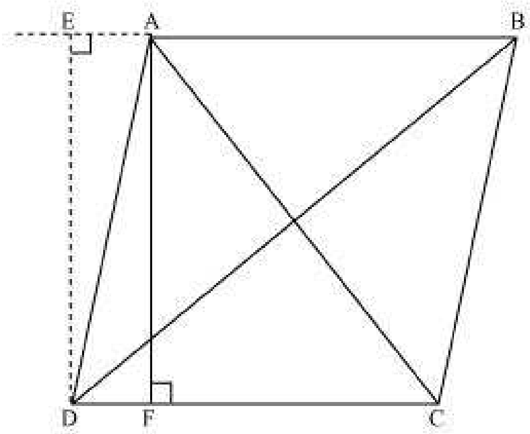
$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer:



Let ABCD be a parallelogram.

Let us draw perpendicular DE on extended side AB, and AF on side DC.

Applying Pythagoras theorem in ΔDEA, we obtain

$DE^2 + EA^2 = DA^2$ ... (*i*)

Applying Pythagoras theorem in ΔDEB, we obtain

$DE^2 + EB^2 = DB^2$

$DE^2 + (EA + AB)^2 = DB^2$

$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$

$DA^2 + AB^2 + 2EA \times AB = DB^2$ ... (*ii*)

Applying Pythagoras theorem in ΔADF, we obtain

$AD^2 = AF^2 + FD^2$

Applying Pythagoras theorem in ΔAFC, we obtain

$AC^2 = AF^2 + FC^2$

$= AF^2 + (DC - FD)^2$

$= AF^2 + DC^2 + FD^2 - 2DC \times FD$

$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$

$AC^2 = AD^2 + DC^2 - 2DC \times FD$ ... (*iii*)

Since ABCD is a parallelogram,

AB = CD … (*iv*)

And, BC = AD … (*v*)

In ΔDEA and ΔADF,

∠DEA = ∠AFD (Both 90°)

∠EAD = ∠ADF (EA || DF)

AD = AD (Common)

∴ ΔEAD ≅ ΔFDA (AAS congruence criterion)

⇒ EA = DF … (vi)

Adding equations (*i*) and (*iii*), we obtain

$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$

$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$

$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$

[Using equations (*iv*) and (*vi*)]

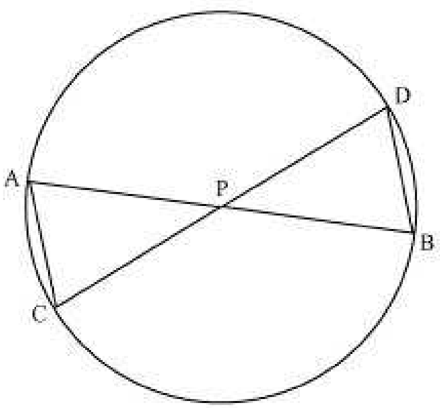
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Question 7:

In the given figure, two chords AB and CD intersect each other at the point P. prove that:

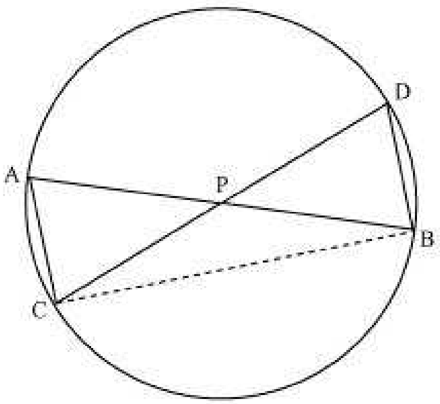
(i) ΔAPC ~ ΔDPB

(ii) AP.BP = CP.DP



Answer:

Let us join CB.



(i) In ΔAPC and ΔDPB,

∠APC = ∠DPB (Vertically opposite angles)

∠CAP = ∠BDP (Angles in the same segment for chord CB)

ΔAPC ~ ΔDPB (By AA similarity criterion)

(ii) We have already proved that

ΔAPC ~ ΔDPB

We know that the corresponding sides of similar triangles are proportional.

$$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$$

$$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$$

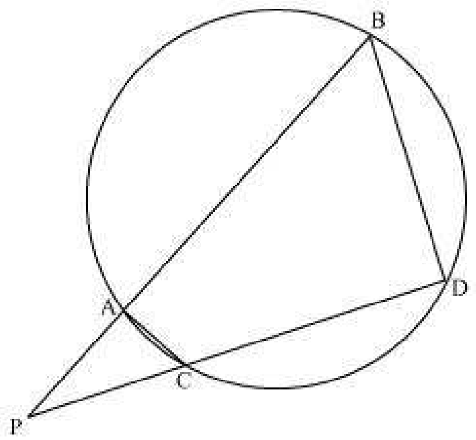
∴ AP. PB = PC. DP

Question 8:

In the given figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) ΔPAC ~ ΔPDB

(ii) PA.PB = PC.PD



Answer:

(i) In ΔPAC and ΔPDB,

∠P = ∠P (Common)

∠PAC = ∠PDB (Exterior angle of a cyclic quadrilateral is ∠PCA = ∠PBD equal to the opposite interior angle)

∴ ΔPAC ~ ΔPDB

(ii)We know that the corresponding sides of similar triangles are proportional.

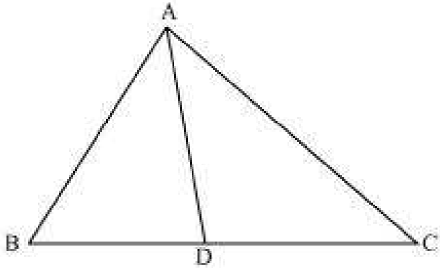
$$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$$

$$\Rightarrow \frac{PA}{PD} = \frac{PC}{PB}$$

∴ PA.PB = PC.PD

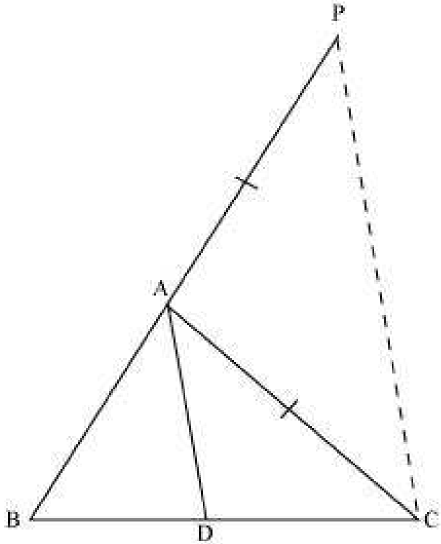
Question 9:

In the given figure, D is a point on side BC of ΔABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of ∠BAC.



Answer:

Let us extend BA to P such that AP = AC. Join PC.



It is given that,

$$\frac{BD}{CD} = \frac{AB}{AC}$$

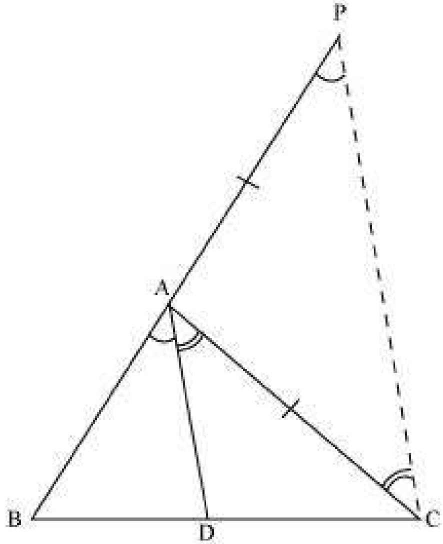
$$\Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$

By using the converse of basic proportionality theorem, we obtain

AD || PC

⇒ ∠BAD = ∠APC (Corresponding angles) … (1)

And, ∠DAC = ∠ACP (Alternate interior angles) … (2)



By construction, we have

AP = AC

⇒ ∠APC = ∠ACP … (3)

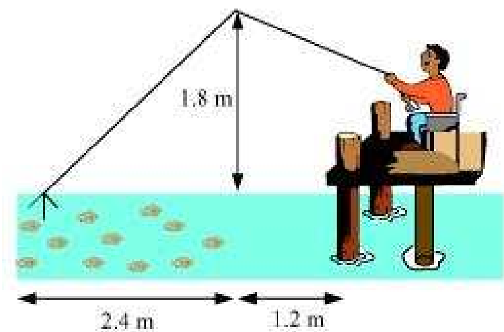
On comparing equations (1), (2), and (3), we obtain

∠BAD = ∠APC

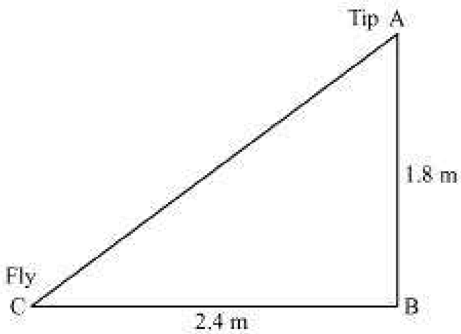
⇒ AD is the bisector of the angle BAC

Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, ho much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Answer:



Let AB be the height of the tip of the fishing rod from the water surface. Let BC be the horizontal distance of the fly from the tip of the fishing rod.

Then, AC is the length of the string.

AC can be found by applying Pythagoras theorem in ΔABC.

$AC^2 = AB^2 + BC^2$

$AB^2 = (1.8\text{ m})^2 + (2.4\text{ m})^2$

$AB^2 = (3.24 + 5.76)\text{ m}^2$

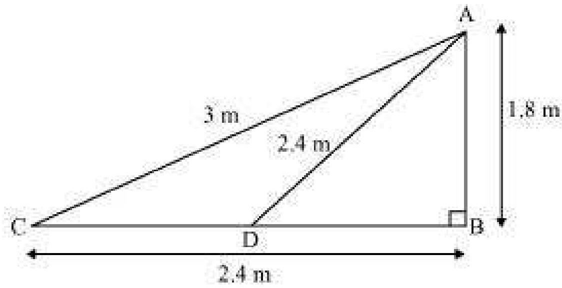
$AB^2 = 9.00\text{ m}^2$

$\Rightarrow AB = \sqrt{9}\text{ m} = 3\text{ m}$

Thus, the length of the string out is 3 m.

She pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = 12 × 5 = 60 cm = 0.6 m



Let the fly be at point D after 12 seconds.

Length of string out after 12 seconds is AD.

AD = AC − String pulled by Nazima in 12 seconds

= (3.00 − 0.6) m

= 2.4 m

In ΔADB,

$AB^2 + BD^2 = AD^2$

$(1.8\text{ m})^2 + BD^2 = (2.4\text{ m})^2$

$BD^2 = (5.76 - 3.24)\text{ m}^2 = 2.52\text{ m}^2$

BD = 1.587 m

Horizontal distance of fly = BD + 1.2 m

= (1.587 + 1.2) m

= 2.787 m

= 2.79 m