Exercise 1.1

Question 1:

Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Answer:

(i) 135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

225 = 135 × 1 + 90

Since remainder 90 ≠ 0, we apply the division lemma to 135 and 90 to obtain

135 = 90 × 1 + 45

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

90 = 2 × 45 + 0

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

(ii)196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

38220 = 196 × 195 + 0

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii)867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

$867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$255 = 102 \times 2 + 51$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$102 = 51 \times 2 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

Question 2:

Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Answer:

Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$,

or $6q + 3$,

or $6q + 5$

Question 3:

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer:

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

Question 4:

Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m.

[**Hint:** Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Answer:

Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$a^2 = (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2$$

$$a^2 = (9q^2) \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4$$

$$= 3 \times (3q^2) \text{ or } 3(3q^2 + 2q) + 1 \text{ or } 3(3q^2 + 4q + 1) + 1$$

$$= 3k_1 \quad \text{or} \quad 3k_2 + 1 \quad \text{or} \quad 3k_3 + 1$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Question 5:

Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Answer:

Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$\therefore a = 3q$ or $3q + 1$ or $3q + 2$

Therefore, every number can be represented as these three forms.

There are three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m,$$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$a^3 = (3q + 1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$a^3 = (3q + 2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Exercise 1.2

Question 1:

Express each number as product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Answer:

(i) $140 = 2\times2\times5\times7 = 2^2\times5\times7$

(ii) $156 = 2\times2\times3\times13 = 2^2\times3\times13$

(iii) $3825 = 3\times3\times5\times5\times17 = 3^2\times5^2\times17$

(iv) $5005 = 5\times7\times11\times13$

(v) $7429 = 17\times19\times23$

Question 2:

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Answer:

(i) 26 and 91

$26 = 2\times13$

$91 = 7\times13$

$\text{HCF} = 13$

$\text{LCM} = 2\times7\times13 = 182$

$\text{Product of the two numbers} = 26\times91 = 2366$

$\text{HCF}\times\text{LCM} = 13\times182 = 2366$

Hence, product of two numbers = HCF × LCM

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$\text{HCF} = 2$

$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

$\text{Product of the two numbers} = 510 \times 92 = 46920$

$$\begin{aligned} \text{HCF} \times \text{LCM} &= 2 \times 23460 \\ &= 46920 \end{aligned}$$

Hence, product of two numbers = HCF × LCM

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$54 = 2 \times 3^3$

$\text{HCF} = 2 \times 3 = 6$

$\text{LCM} = 2^4 \times 3^3 \times 7 = 3024$

$\text{Product of the two numbers} = 336 \times 54 = 18144$

$\text{HCF} \times \text{LCM} = 6 \times 3024 = 18144$

Hence, product of two numbers = HCF × LCM

Question 3:

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Answer:

(i) 12,15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$\text{HCF} = 3$

$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

$\text{HCF} = 1$

$\text{LCM} = 17 \times 23 \times 29 = 11339$

(iii) 8,9 and 25

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

$\text{HCF} = 1$

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

Question 4:

Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer:

$\text{HCF}(306, 657) = 9$

We know that, $\text{LCM} \times \text{HCF} = $ Product of two numbers

$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$\text{LCM} = 22338$

Question 5:

Check whether 6^n can end with the digit 0 for any natural number *n*.

Answer:

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as 10 = 2 × 5

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of *n*, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number *n*.

Question 6:

Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Answer:

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Question 7:

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer:

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$18 = 2 \times 3 \times 3$

And, $12 = 2 \times 2 \times 3$

LCM of 12 and 18 = $2 \times 2 \times 3 \times 3 = 36$

Therefore, Ravi and Sonia will meet together at the starting pointafter 36 minutes.

Exercise 1.3

Question 1:

Prove that $\sqrt{5}$ is irrational.

Answer:

Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers *a*, *b* ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Let *a* and *b* have a common factor other than 1. Then we can divide them by the common factor, and assume that *a* and *b* are co-prime.

$$a = \sqrt{5}b$$
$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that *a* is divisible by 5.

Let $a = 5k$, where *k* is an integer

$$(5k)^2 = 5b^2$$

$b^2 = 5k^2$ This means that b^2 is divisible by 5 and hence, *b* is divisible by 5.

This implies that *a* and *b* have 5 as a common factor.

And this is a contradiction to the fact that *a* and *b* are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Question 2:

Prove that $3+2\sqrt{5}$ is irrational.

Answer:

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers *a*, *b* ($b \neq 0$) such that

$$3+2\sqrt{5}=\frac{a}{b}$$

$$2\sqrt{5}=\frac{a}{b}-3$$

$$\sqrt{5}=\frac{1}{2}\left(\frac{a}{b}-3\right)$$

Since *a* and *b* are integers, $\frac{1}{2}\left(\frac{a}{b}-3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false. Therefore, $3+2\sqrt{5}$ is irrational.

Question 3:

Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6+\sqrt{2}$

Answer:

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers *a*, *b* ($b \neq 0$) such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\sqrt{2} = \frac{b}{a}$$

$\frac{b}{a}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two integers *a*, *b* ($b \neq 0$) such that

$7\sqrt{5} = \frac{a}{b}$ for some integers *a* and *b*

$$\therefore \sqrt{5} = \frac{a}{7b}$$

$\frac{a}{7b}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6+\sqrt{2}$

Let $6+\sqrt{2}$ be rational.

Therefore, we can find two integers *a*, *b* ($b \neq 0$) such that

$$6+\sqrt{2}=\frac{a}{b}$$

$$\sqrt{2}=\frac{a}{b}-6$$

Since *a* and *b* are integers, $\frac{a}{b}-6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is false and hence, $6+\sqrt{2}$ is irrational.

Exercise 1.4

Question 1:

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Answer:

(i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455 = 5 \times 7 \times 13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$1600 = 2^6 \times 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$343 = 7^3$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 \times 5^2}$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15}=\frac{2\times 3}{3\times 5}=\frac{2}{5}$

The denominator is of the form 5^n.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50}=\frac{7\times 5}{10\times 5}=\frac{7}{10}$

$10=2\times 5$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210}=\frac{11\times 7}{30\times 7}=\frac{11}{30}$

$30=2\times 3\times 5$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 3 as its factors, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

Question 2:

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer:

(i) $\frac{13}{3125} = 0.00416$

$$
\begin{array}{r}
0.00416 \\
3125\overline{)13.00000} \\
0 \\
\hline
130 \\
0 \\
\hline
1300 \\
0 \\
\hline
13000 \\
12500 \\
\hline
5000 \\
3125 \\
\hline
18750 \\
18750 \\
\hline
\times \\
\hline
\end{array}
$$

(ii) $\frac{17}{8} = 2.125$

$$
\begin{array}{r}
2.125 \\
8\overline{)\ 17} \\
16 \\
\hline
10 \\
8 \\
\hline
20 \\
16 \\
\hline
40 \\
40 \\
\hline
\times \\
\hline
\end{array}
$$

(iv) $\frac{15}{1600} = 0.009375$

$$\begin{array}{r}
0.009375 \\
1600\overline{)15.000000} \\
0 \\ \hline
150 \\
0 \\ \hline
1500 \\
0 \\ \hline
15000 \\
14400 \\ \hline
6000 \\
\underline{4800} \\
12000 \\
\underline{11200} \\
8000 \\
8000 \\ \hline
\times \\ \hline
\end{array}$$

(vi) $\frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$

$$\begin{array}{r}
0.115 \\
200\overline{)23.000} \\
0 \\ \hline
230 \\
\underline{200} \\
300 \\
\underline{200} \\
1000 \\
\underline{1000} \\
\times \\ \hline
\end{array}$$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ 0 \\ \hline 20 \\ 20 \\ \hline \times \\ \hline \end{array}$$

(viii) $\frac{6}{15} = \frac{2\times 3}{3\times 5} = \frac{2}{5} = 0.4$

(ix) $\frac{35}{50} = 0.7$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ 0 \\ \hline 350 \\ 350 \\ \hline \times \\ \hline \end{array}$$

Question 3:

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of q?

(i) 43.123456789 (ii) 0.120120012000120000… (iii) $43.\overline{123456789}$

Answer:

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000 …

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.