Exercise 8.1

Question 1:

In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A

(ii) sin C, cos C

Answer:

Applying Pythagoras theorem for ΔABC, we obtain

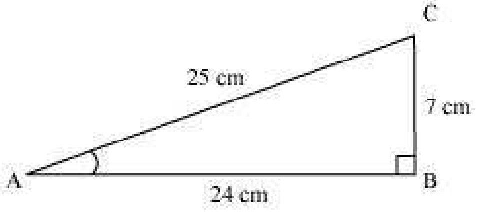
$AC^2 = AB^2 + BC^2$

$= (24 \text{ cm})^2 + (7 \text{ cm})^2$

$= (576 + 49) \text{ cm}^2$

$= 625 \text{ cm}^2$

$\therefore$ AC = $\sqrt{625}$ cm = 25 cm

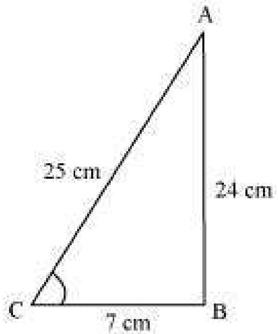


(i) sin A = $\dfrac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

$= \dfrac{7}{25}$

cos A = $\dfrac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{24}{25}$

(ii)



sin C = $\frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC}$

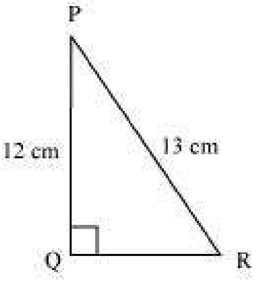
$= \frac{24}{25}$

cos C = $\frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC}$

$= \frac{7}{25}$

Question 2:

In the given figure find tan P − cot R



Answer:

Applying Pythagoras theorem for ΔPQR, we obtain

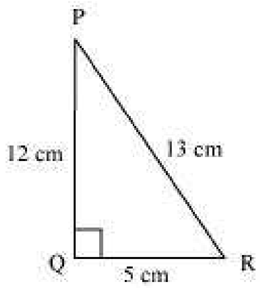
$PR^2 = PQ^2 + QR^2$

$(13 \text{ cm})^2 = (12 \text{ cm})^2 + QR^2$

$169\ cm^2 = 144\ cm^2 + QR^2$

$25\ cm^2 = QR^2$

$QR = 5$ cm



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

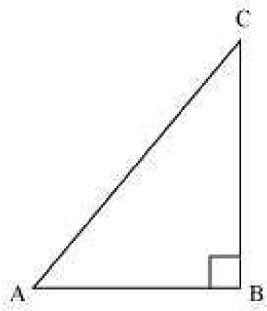
tan P − cot R = $\frac{5}{12} - \frac{5}{12} = 0$

Question 3:

If sin A = $\frac{3}{4}$, calculate cos A and tan A.

Answer:

Let ΔABC be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be 3*k*. Therefore, AC will be 4*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

AB = $\sqrt{7}k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

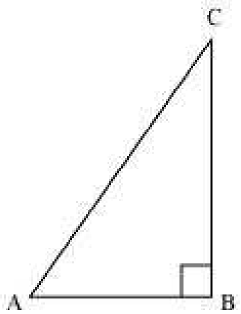
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Question 4:

Given 15 cot A = 8. Find sin A and sec A

Answer:

Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

It is given that,

$\cot A = \frac{8}{15}$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be $8k$.Therefore, BC will be $15k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

$AC = 17k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

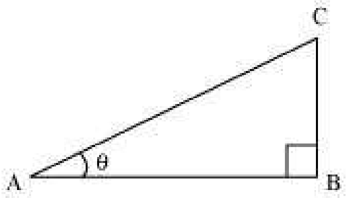
$$= \frac{AC}{AB} = \frac{17}{8}$$

Question 5:

Given sec θ = $\frac{13}{12}$, calculate all other trigonometric ratios.

Answer:

Consider a right-angle triangle ΔABC, right-angled at point B.



$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle\theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is 13*k*, AB will be 12*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

BC = 5*k*

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{\text{AB}}{\text{AC}} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \frac{\text{BC}}{\text{AB}} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{\text{AB}}{\text{BC}} = \frac{12k}{5k} = \frac{12}{5}$$

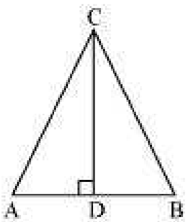
$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle\theta} = \frac{\text{AC}}{\text{BC}} = \frac{13k}{5k} = \frac{13}{5}$$

Question 6:

If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Answer:

Let us consider a triangle ABC in which CD ⊥ AB.

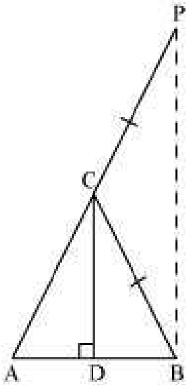


It is given that

cos A = cos B

$$\Rightarrow \frac{\text{AD}}{\text{AC}} = \frac{\text{BD}}{\text{BC}} \quad \ldots (1)$$

We have to prove ∠A = ∠B. To prove this, let us extend AC to P such that BC = CP.



From equation (1), we obtain

$$\frac{AD}{BD} = \frac{AC}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{CP} \qquad \text{(By construction, we have } BC = CP) \qquad \ldots (2)$$

By using the converse of B.P.T,

CD||BP

⇒∠ACD = ∠CPB (Corresponding angles) … (3)

And, ∠BCD = ∠CBP (Alternate interior angles) … (4)

By construction, we have BC = CP.

∴ ∠CBP **=** ∠CPB (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

∠ACD = ∠BCD … (6)

In ΔCAD and ΔCBD,

∠ACD = ∠BCD [Using equation (6)]

∠CDA = ∠CDB [Both 90°]

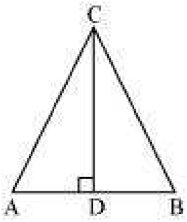
Therefore, the remaining angles should be equal.

∴∠CAD = ∠CBD

⇒ ∠A = ∠B

Alternatively,

Let us consider a triangle ABC in which CD $\perp$ AB.



It is given that,

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

Let $\frac{AD}{BD} = \frac{AC}{BC} = k$

$\Rightarrow$ AD = k BD … (1)

And, AC = k BC … (2)

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$CD^2 = AC^2 - AD^2$ … (3)

And, $CD^2 = BC^2 - BD^2$ … (4)

From equations (3) and (4), we obtain

$AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (k\ BC)^2 - (k\ BD)^2 = BC^2 - BD^2$

$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1$

$\Rightarrow k = 1$

Putting this value in equation (2), we obtain

AC = BC

$\Rightarrow \angle A = \angle B$(Angles opposite to equal sides of a triangle)

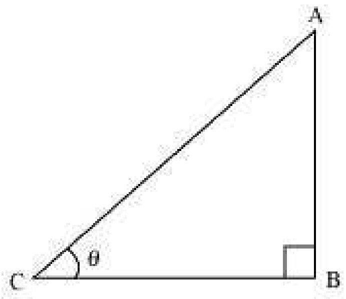
Question 7:

If $\cot\theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Answer:

Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

If BC is $7k$, then AB will be $8k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

AC = $\sqrt{113}k$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Question 8:

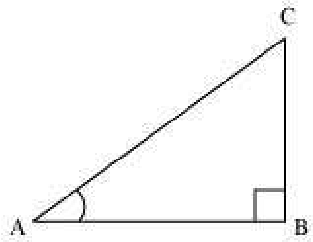
If 3 cot A = 4, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer:

It is given that 3cot A = 4

Or, cot A = $\frac{4}{3}$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is $4k$, then BC will be $3k$, where k is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

AC = $5k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4k}{5k} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3k}{5k} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}=\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$=\frac{\frac{7}{16}}{\frac{25}{16}}=\frac{7}{25}$$

$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2$

$$=\frac{16}{25}-\frac{9}{25}=\frac{7}{25}$$

$$\therefore\ \frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$$

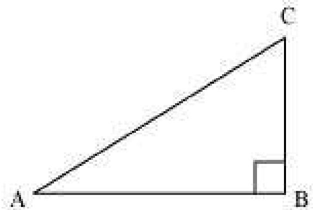
Question 9:

In ΔABC, right angled at B. If $\tan A=\frac{1}{\sqrt{3}}$, find the value of

(i) sin A cos C + cos A sin C

(ii) cos A cos C − sin A sin C

Answer:



$$\tan A = \frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

If BC is *k*, then AB will be $\sqrt{3}k$, where *k* is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}k\right)^2 + (k)^2$

$= 3k^2 + k^2 = 4k^2$

∴ AC = $2k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) cos A cos C − sin A sin C

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Question 10:

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

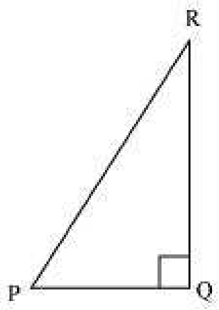
Answer:

Given that, PR + QR = 25

PQ = 5

Let PR be x.

Therefore, QR = 25 − x



Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

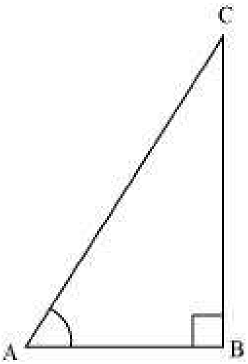
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

(v) sin θ = $\frac{4}{3}$, for some angle θ

Answer:

(i) Consider a ΔABC, right-angled at B.



$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{12}{5}$$

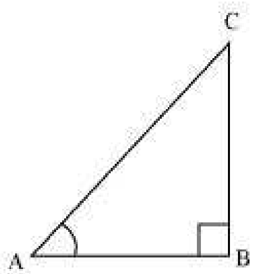
But $\frac{12}{5} > 1$

$\therefore \tan A > 1$

So, $\tan A < 1$ is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be $12k$, AB will be $5k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

$BC = 10.9k$

It can be observed that for given two sides $AC = 12k$ and $AB = 5k$,

BC should be such that,

$AC - AB < BC < AC + AB$

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17\,k$

However, BC = $10.9k$. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of $\angle$A.

Hence, the given statement is false.

(v) $\sin\theta = \frac{4}{3}$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible.

Hence, the given statement is false

Exercise 8.2

Question 1:

Evaluate the following

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Answer:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\left(2 + 2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}}$$

$$= \frac{\sqrt{3}\left(2\sqrt{6} - 2\sqrt{2}\right)}{\left(2\sqrt{6} + 2\sqrt{2}\right)\left(2\sqrt{6} - 2\sqrt{2}\right)}$$

$$= \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{\left(2\sqrt{6}\right)^2 - \left(2\sqrt{2}\right)^2} = \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{24 - 8} = \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{16}$$

$$= \frac{\sqrt{18} - \sqrt{6}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

$$=\frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}=\frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$=\frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}}=\frac{\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)}$$

$$=\frac{\left(3\sqrt{3}-4\right)\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)\left(3\sqrt{3}-4\right)}=\frac{\left(3\sqrt{3}-4\right)^2}{\left(3\sqrt{3}\right)^2-(4)^2}$$

$$=\frac{27+16-24\sqrt{3}}{27-16}=\frac{43-24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60°+4\sec^2 30°-\tan^2 45°}{\sin^2 30°+\cos^2 30°}$

$$=\frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$=\frac{5\left(\frac{1}{4}\right)+\left(\frac{16}{3}\right)-1}{\frac{1}{4}+\frac{3}{4}}$$

$$=\frac{\frac{15+64-12}{12}}{\frac{4}{4}}=\frac{67}{12}$$

Question 2:

Choose the correct option and justify your choice.

(i) $\frac{2\tan 30°}{1+\tan^2 30°} =$

(A). sin60°

(B). cos60°

(C). tan60°

(D). sin30°

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°} =$

(A). tan90°

(B). 1

(C). sin45°

(D). 0

(iii) sin2A = 2sinA is true when A =

(A). 0°

(B). 30°

(C). 45°

(D). 60°

(iv) $\frac{2\tan 30°}{1-\tan^2 30°} =$

(A). cos60°

(B). sin60°

(C). tan60°

(D). sin30°

Answer:

(i) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$

$$= \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

Hence, (D) is correct.

(iii)Out of the given alternatives, only A = 0° is correct.

As sin 2A = sin 0° = 0

2 sinA = 2sin 0° = 2(0) = 0

Hence, (A) is correct.

(iv) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \sqrt{3}$$

Out of the given alternatives, only tan 60° $= \sqrt{3}$

Hence, (C) is correct.

Question 3:

If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$;

0° < A + B ≤ 90°, A > B find A and B.

Answer:

$\tan(A+B) = \sqrt{3}$

$\Rightarrow \tan(A+B) = \tan 60$

$\Rightarrow$ A + B = 60 … (1)

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow$ tan (A − B) = tan30

$\Rightarrow$ A − B = 30 … (2)

On adding both equations, we obtain

2A = 90

$\Rightarrow$ A = 45

From equation (1), we obtain

45 + B = 60

B = 15

Therefore, ∠A = 45° and ∠B = 15°

Question 4:

State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B

(ii) The value of $\sin\theta$ increases as θ increases

(iii) The value of $\cos \theta$ increases as θ increases

(iv) $\sin\theta = \cos \theta$ for all values of θ

(v) $\cot A$ is not defined for $A = 0°$

Answer:

(i) $\sin (A + B) = \sin A + \sin B$

Let $A = 30°$ and $B = 60°$

$\sin (A + B) = \sin (30° + 60°)$

$= \sin 90°$

$= 1$

$\sin A + \sin B = \sin 30° + \sin 60°$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

Clearly, $\sin (A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of $\sin \theta$ increases as θ increases in the interval of $0° < \theta < 90°$ as

$\sin 0° = 0$

$$\sin 30° = \frac{1}{2} = 0.5$$

$$\sin 45° = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$$

$\sin 90° = 1$

Hence, the given statement is true.

(iii) $\cos 0° = 1$

$$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60° = \frac{1}{2} = 0.5$$

cos90° = 0

It can be observed that the value of cos θ does not increase in the interval of 0° < θ < 90°.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As $\sin 45° = \frac{1}{\sqrt{2}}$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

It is not true for all other values of θ.

As $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$,

Hence, the given statement is false.

(v) cot A is not defined for A = 0°

As $\cot A = \frac{\cos A}{\sin A}$,

$\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ = undefined

Hence, the given statement is true.

Exercise 8.3

Question 1:

Evaluate

(I) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(III) cos 48° − sin 42°

(IV)cosec 31° − sec 59°

Answer:

(I) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$

$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$

$= \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

(III)cos 48° − sin 42° = cos (90°− 42°) − sin 42°

= sin 42° − sin 42°

= 0

(IV) cosec 31° − sec 59° = cosec (90° − 59°) − sec 59°

= sec 59° − sec 59°

= 0

Question 2:

Show that

(I) tan 48° tan 23° tan 42° tan 67° = 1

(II)cos 38° cos 52° − sin 38° sin 52° = 0

Answer:

(I) $\tan 48° \tan 23° \tan 42° \tan 67°$

$= \tan (90° - 42°) \tan (90° - 67°) \tan 42° \tan 67°$

$= \cot 42° \cot 67° \tan 42° \tan 67°$

$= (\cot 42° \tan 42°) (\cot 67° \tan 67°)$

$= (1) (1)$

$= 1$

(II) $\cos 38° \cos 52° - \sin 38° \sin 52°$

$= \cos (90° - 52°) \cos (90°-38°) - \sin 38° \sin 52°$

$= \sin 52° \sin 38° - \sin 38° \sin 52°$

$= 0$

Question 3:

If $\tan 2A = \cot (A - 18°)$, where $2A$ is an acute angle, find the value of A.

Answer:

Given that,

$\tan 2A = \cot (A - 18°)$

$\cot (90° - 2A) = \cot (A - 18°)$

$90° - 2A = A - 18°$

$108° = 3A$

$A = 36°$

Question 4:

If $\tan A = \cot B$, prove that $A + B = 90°$

Answer:

Given that,

$\tan A = \cot B$

$\tan A = \tan (90° - B)$

$A = 90° - B$

A + B = 90°

Question 5:

If sec 4A = cosec (A− 20°), where 4A is an acute angle, find the value of A.

Answer:

Given that,

sec 4A = cosec (A − 20°)

cosec (90° − 4A) = cosec (A − 20°)

90° − 4A= A− 20°

110° = 5A

A = 22°

Question 6:

If A, Band C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Answer:

We know that for a triangle ABC,

∠ A + ∠B + ∠C = 180°

∠B + ∠C= 180° − ∠A

$$\frac{\angle B+\angle C}{2} = 90° - \frac{\angle A}{2}$$

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$

$$= \cos\left(\frac{A}{2}\right)$$

Question 7:

Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Answer:

sin 67° + cos 75°

$= \sin (90° - 23°) + \cos (90° - 15°)$

$= \cos 23° + \sin 15°$

Exercise 8.4

Question 1:

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Answer:

We know that,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$$\sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

$\sqrt{1 + \cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore, $\sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$

We know that, $\tan A = \frac{\sin A}{\cos A}$

However, $\cot A = \frac{\cos A}{\sin A}$

Therefore, $\tan A = \frac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Question 2:

Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer:

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\text{cosec } A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Question 3:

Evaluate

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) sin25° cos65° + cos25° sin65°

Answer:

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$= \frac{[\sin(90° - 27°)]^2 + \sin^2 27°}{[\cos(90° - 73°)]^2 + \cos^2 73°}$

$= \frac{[\cos 27°]^2 + \sin^2 27°}{[\sin 73°]^2 + \cos^2 73°}$

$= \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°}$

$= \frac{1}{1}$ (As $\sin^2 A + \cos^2 A = 1$)

= 1

(ii) sin25° cos65° + cos25° sin65°

$= (\sin 25°)\{\cos(90° - 25°)\} + \cos 25° \{\sin(90° - 25°)\}$

$= (\sin 25°)(\sin 25°) + (\cos 25°)(\cos 25°)$

$= \sin^2 25° + \cos^2 25°$

= 1 (As $\sin^2 A + \cos^2 A = 1$)

Question 4:

Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

(A) 0

(B) 1

(C) 2

(D) −1

(iii) (secA + tanA) (1 − sinA) =

(A) secA

(B) sinA

(C) cosecA

(D) cosA

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$

(B) −1

(C) $\cot^2 A$

(D) $\tan^2 A$

Answer:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9(1)$ [As $\sec^2 A - \tan^2 A = 1$]

$= 9$

Hence, alternative (B) is correct.

(ii)

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$=\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$=\frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$=\frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}=2$$

Hence, alternative (C) is correct.

(iii) (secA + tanA) (1 − sinA)

$$=\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$=\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$=\frac{1-\sin^2 A}{\cos A}=\frac{\cos^2 A}{\cos A}$$

= cosA

Hence, alternative (D) is correct.

(iv) $$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$$

$$=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer:

(i) $(\text{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.}=(\text{cosec}\,\theta-\cot\theta)^2$$

$$=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$

$$=\frac{(1-\cos\theta)^2}{(\sin\theta)^2}=\frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}=\frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}$$

$$=\text{R.H.S.}$$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$

$$= \text{R.H.S.}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta$

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$= \frac{(1+\sin\theta\ \cos\theta)}{(\sin\theta\ \cos\theta)}$$

= secθ cosec θ +

= R.H.S.

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$$\text{L.H.S.} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}} = (\cos A + 1)$$

$$= \frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A}$$

= R.H.S

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$,

L.H.S = $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

= cosec A + cot A

= R.H.S

(vi) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

$$\text{L.H.S.} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} \qquad = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} \qquad = \sec A + \tan A$$

$$= \text{R.H.S.}$$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$

$$= \frac{\sin\theta \times \left(1-2\sin^2\theta\right)}{\cos\theta \times \left\{2\left(1-\sin^2\theta\right)-1\right\}}$$

$$= \frac{\sin\theta \times \left(1-2\sin^2\theta\right)}{\cos\theta \times \left(1-2\sin^2\theta\right)}$$

$$= \tan\theta = \text{R.H.S}$$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$
\begin{aligned}
\text{L.H.S} &= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\
&= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A \\
&= (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A + \sec^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right) \\
&= (1) + (1 + \cot^2 A + 1 + \tan^2 A) + (2) + (2) \\
&= 7 + \tan^2 A + \cot^2 A \\
&= \text{R.H.S}
\end{aligned}
$$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

$$
\begin{aligned}
\text{L.H.S} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \\
&= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right) \\
&= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right) \\
&= \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A} \\
&= \sin A \cos A
\end{aligned}
$$

$$
\begin{aligned}
\text{R.H.S} &= \frac{1}{\tan A + \cot A} \\
&= \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}} = \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \\
&= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A
\end{aligned}
$$

Hence, L.H.S = R.H.S

(x) $\left(\dfrac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\dfrac{\sin^2 A}{\cos^2 A}}{1+\dfrac{\cos^2 A}{\sin^2 A}} = \frac{\dfrac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\dfrac{\sin^2 A+\cos^2 A}{\sin^2 A}}$$

$$= \frac{\dfrac{1}{\cos^2 A}}{\dfrac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \frac{1+\tan^2 A - 2\tan A}{1+\cot^2 A - 2\cot A}$$

$$= \frac{\sec^2 A - 2\tan A}{\operatorname{cosec}^2 A - 2\cot A}$$

$$= \frac{\dfrac{1}{\cos^2 A} - \dfrac{2\sin A}{\cos A}}{\dfrac{1}{\sin^2 A} - \dfrac{2\cos A}{\sin A}} = \frac{\dfrac{1-2\sin A\cos A}{\cos^2 A}}{\dfrac{1-2\sin A\cos A}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$