PRE BOARD EXAMINATION (2017-18)

CLASS: XII

SUB: MATHEMATICS

TimeAllowed:3 hours Max.Marks:100

General Instructions:

1. *All questions are compulsory*
2. *This question paper consists of 29 questions divided into 4 sections A, B,C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of 2 marks each, section C comprises of 11 questions of 4 marks each and section D comprises of 6 questions of 6 marks each*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question*
4. *There is no overall choice. However, internal choice has been provided in questions. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted. You may ask for logarithmic tables, if required*

SECTION A

1) If f: $R \to R$ be given by $f(x) = x^2 + 3$ and $g(x) = \sqrt{x-3}$, then find f og (x)

2) Evaluate $\tan^{-1}(\tan\frac{2\pi}{3})$

3) If A is a non singular matrix of order 3×3 and $|A| = 9$, Find $|adjA|$

4) Write the distance of a plane $2x + 3y + 4z = 9$ from the origin..

SECTION B

5) Show that the function $f: R \to R$ defined by $f(x) = \frac{2x-1}{3}, x \in R$ is one-one and onto Function.

6) Find point of discontinuity

$$f(x) = \begin{cases} x^3 - 3, x \le 2 \\ x^2 + 1, x > 2 \end{cases}$$

7) Using differentials find the approximate value of $(82)^{\frac{1}{2}}$

8) Find the sum of order and degree of the following diff. equation

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1+x) = 0$$

9) Find a unit vector perpendicular to the vectors $3\hat{i} + 2\hat{j} - \hat{k}$ and $-2\hat{i} + \hat{j} + \hat{k}$

Taking $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$

10) Find the vector and the cartesian equations of the line through the point (5, 2, – 4) and which is parallel to the vector 3i +2j -8k .

11) The probabilities of A,B and C solving a problem are 1/2,1/3 and 1/4 respectively. If the problem is attempted by all simultaneously, find the probability of exactly one of them solving it

12) Represent the following LPP Graphically

MaxZ=3x+4y

Subject to constraints $x+y \leq 4$, $x,y \geq 0$

SECTION C

13) Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^2$

OR

Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$$

14) Show that $(a^2-b^2)y\frac{d^2y}{dx^2}+b^2=0$ if x=a sinpt, y=b cospt

15) Verify Rolle's theorem for $f(x) = x^2 + 2$, at [-2,2]

16) Evaluate $\int \frac{Sin\,x}{Sin(x-a)}dx$

17) Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

18) Show that the differential equation $(x-y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it

19) Prove that $\vec{a}.\left\{\left(\vec{b}\times\vec{c}\right)\times\left(\vec{a}+2\vec{b}+3\vec{c}\right)\right\} = \left[\vec{a},\vec{b},\vec{c}\right]$

20) Find the equation of the plane through the line of intersection of the planes $x+y+z=1$, $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$.

21) A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six .

22) Two cards are drawn successively, without replacement, from a well shuffled pack of 52 cards. Find the probability distribution of number spades.

23) There are two factories located one at place P and the other at place Q. From these Locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

From/To	Cost (In Rs)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. What will be the minimum transportation cost

OR

A manufacturer has three machines I, IIand III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for atleast 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each iten should she produce so as to maximise her profit assuming that she can sell all the items that sh produced? What will be the maximum profit?

SECTION D

24) Using matrix method , solve the equations

$$2x + 6y = 2\ ,\ 3x - z = -8\ ,\ 2x - y + z = -3$$

OR

Find the inverse of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, using elementary row transformation

25) An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A Soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance. Write two values of Indian Army in your life.

OR

An open topped box is to be constructed by removing equal squares from each corner of a 3 meter by 8 meter rectangular sheet of aluminum and folding up the sides. Find the volume of the largest such box.

26) Evaluate $\int_0^{\pi} \log \sin x \, dx$

27) Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3) .Also find eq. of line .

OR

Find the distance of the point (-2, 3,-4) from the line

$\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$, measured parallel to the plane 4x + 12 y -3z + 1 = 0

28) a.) *Find the value of a and b such that the function defind by*

If $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & \text{if } x < 4 \\ a+b & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + 2b & \text{if } x > 4 \end{cases}$ *is a continous function at* $x = 4$

b. If $y = [\log(x + \sqrt{x^2+1})]^2$, *Show that* $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$

29) Let f: $[0, \infty) \to R$ be a function defined by $f(x)=9x^2+6x-5$. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse.

OR

Let * be a binary operation defined on Q x Q by (a, b)*(c, d) = (a c ,b +ad), where Q is the set of rational number. Determine, whether * is commutative and associative. Find the identity element for * and the invertible elements of Q x Q