CHAPTER 2

POLYNOMIALS

(A) Main Concepts and Results

- Geometrical meaning of zeroes of a polynomial: The zeroes of a polynomial $p(x)$ are precisely the x-coordinates of the points where the graph of $y = p(x)$ intersects the x-axis.
- Relation between the zeroes and coefficients of a polynomial: If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$.
- If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then $\alpha+\beta+\gamma = -\frac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ and $\alpha\beta\gamma = \frac{-d}{a}$.
- The division algorithm states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x)\, q(x) + r(x)$, where $r(x) = 0$ or degree $r(x)$ < degree $g(x)$.

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1: If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(A) 10 (B) –10 (C) 5 (D) –5

Solution : Answer (B)

Sample Question 2: Given that two of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, the third zero is

(A) $\frac{-b}{a}$ (B) $\frac{b}{a}$ (C) $\frac{c}{a}$ (D) $-\frac{d}{a}$

Solution : Answer (A). [**Hint**: Because if third zero is α, sum of the zeroes $= \alpha + 0 + 0 = \frac{-b}{a}$]

EXERCISE 2.1

Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial $(k–1)\, x^2 + k\, x + 1$ is –3, then the value of k is

(A) $\frac{4}{3}$ (B) $\frac{-4}{3}$ (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

2. A quadratic polynomial, whose zeroes are –3 and 4, is

(A) $x^2 – x + 12$ (B) $x^2 + x + 12$

(C) $\frac{x^2}{2} - \frac{x}{2} - 6$ (D) $2x^2 + 2x –24$

3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)\, x + b$ are 2 and –3, then

(A) $a = –7, b = –1$ (B) $a = 5, b = –1$

(C) $a = 2, b = – 6$ (D) $a = 0, b = – 6$

4. The number of polynomials having zeroes as –2 and 5 is

(A) 1 (B) 2 (C) 3 (D) more than 3

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(A) $-\frac{c}{a}$ (B) $\frac{c}{a}$ (C) 0 (D) $-\frac{b}{a}$

6. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is

(A) $b – a + 1$ (B) $b – a – 1$ (C) $a – b + 1$ (D) $a – b –1$

7. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(A) both positive (B) both negative

(C) one positive and one negative (D) both equal

8. The zeroes of the quadratic polynomial $x^2 + kx + k$, $k \neq 0$,

(A) cannot both be positive (B) cannot both be negative

(C) are always unequal (D) are always equal

9. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then

(A) c and a have opposite signs (B) c and b have opposite signs

(C) c and a have the same sign (D) c and b have the same sign

10. If one of the zeroes of a quadratic polynomial of the form $x^2+ax + b$ is the negative of the other, then it

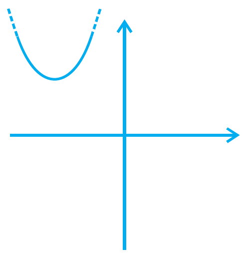
(A) has no linear term and the constant term is negative.

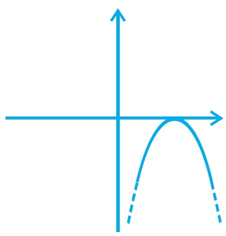
(B) has no linear term and the constant term is positive.

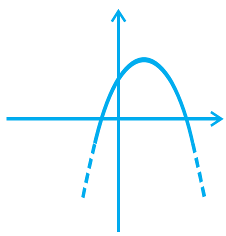
(C) can have a linear term but the constant term is negative.

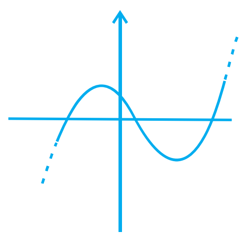
(D) can have a linear term but the constant term is positive.

11. Which of the following is not the graph of a quadratic polynomial?

(A)


(B)


(C)


(D)


(C) Short Answer Questions with Reasoning

Sample Question 1: Can $x - 1$ be the remainder on division of a polynomial $p(x)$ by $2x + 3$? Justify your answer.

Solution : No, since degree $(x - 1) = 1 =$ degree $(2x + 3)$.

Sample Question 2: Is the following statement True or False? Justify your answer.

If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both negative, then a, b and c all have the same sign.

Solution : True, because $-\frac{b}{a}$ = sum of the zeroes < 0, so that $\frac{b}{a} > 0$. Also the product of the zeroes $= \frac{c}{a} > 0$.

EXERCISE 2.2

1. Answer the following and justify:

(i) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5?

(ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s$, $p \neq 0$?

(iii) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

(iv) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

(v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

2. Are the following statements 'True' or 'False'? Justify your answers.

(i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

(ii) If the graph of a polynomial intersects the x-axis at only one point, it cannot be a quadratic polynomial.

(iii) If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.

(iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.

(vi) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b and c is non-negative.

(vii) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeros is $\frac{1}{2}$

(D) Short Answer Questions

Sample Question 1: Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$, and verify the relation between the coefficients and the zeroes of the polynomial.

Solution : $x^2 + \frac{1}{6}x - 2 = \frac{1}{6}(6x^2 + x - 12) = \frac{1}{6}[6x^2 + 9x - 8x - 12]$

$$= \frac{1}{6}[3x(2x + 3) - 4(2x + 3)] = \frac{1}{6}(3x - 4)(2x + 3)$$

Hence, $\frac{4}{3}$ and $-\frac{3}{2}$ are the zeroes of the given polynomial.

The given polynomial is $x^2 + \frac{1}{6}x - 2$.

The sum of zeroes $= \frac{4}{3} + \left(-\frac{3}{2}\right) = \frac{-1}{6} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ and

the product of zeroes $= \frac{4}{3} \times \left(\frac{-3}{2}\right) = -2 = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

EXERCISE 2.3

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials:

1. $4x^2 - 3x - 1$ **2.** $3x^2 + 4x - 4$

3. $5t^2 + 12t + 7$

4. $t^3 - 2t^2 - 15t$

5. $2x^2 + \frac{7}{2}x + \frac{3}{4}$

6. $4x^2 + 5\sqrt{2}x - 3$

7. $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

8. $v^2 + 4\sqrt{3}v - 15$

9. $y^2 + \frac{3}{2}\sqrt{5}y - 5$

10. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

(E) Long Answer Questions

Sample Question 1: Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $-\frac{3}{2}$, respectively. Also find its zeroes.

Solution : A quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $-\frac{3}{2}$ is $x^2 - \sqrt{2}x - \frac{3}{2}$

$$x^2 - \sqrt{2}x - \frac{3}{2} = \frac{1}{2}[2x^2 - 2\sqrt{2}x - 3]$$

$$= \frac{1}{2}[2x^2 + \sqrt{2}x - 3\sqrt{2x} - 3]$$

$$= \frac{1}{2}[\sqrt{2}x(\sqrt{2}x + 1) - 3(\sqrt{2}x + 1)]$$

$$= \frac{1}{2}[\sqrt{2}x + 1][\sqrt{2}x - 3]$$

Hence, the zeroes are $-\frac{1}{\sqrt{2}}$ and $\frac{3}{\sqrt{2}}$.

Sample Question 2: If the remainder on division of $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

Solution : Let $p(x) = x^3 + 2x^2 + kx + 3$

Then, $p(3) = 3^3 + 2 \times 3^2 + 3k + 3 = 21$

i.e., $3k = -27$

i.e., $k = -9$

Hence, the given polynomial will become $x^3 + 2x^2 - 9x + 3$.

Now,

$$
\begin{array}{r|l}
x-3 & x^3 + 2x^2 - 9x + 3 \;(x^2 + 5x + 6 \\
 & \underline{x^3 - 3x^2} \\
 & \quad 5x^2 - 9x + 3 \\
 & \quad \underline{5x^2 - 15x} \\
 & \qquad 6x + 3 \\
 & \qquad \underline{6x - 18} \\
 & \qquad\quad 21
\end{array}
$$

So, $x^3 + 2x^2 - 9x + 3 = (x^2 + 5x + 6)(x - 3) + 21$

i.e., $x^3 + 2x^2 - 9x - 18 = (x - 3)(x^2 + 5x + 6)$

$= (x - 3)(x + 2)(x + 3)$

So, the zeroes of $x^3 + 2x^2 + kx - 18$ are $3, -2, -3$.

EXERCISE 2.4

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

(i) $\frac{-8}{3}, \frac{4}{3}$ (ii) $\frac{21}{8}, \frac{5}{16}$

(iii) $-2\sqrt{3}, -9$ (iv) $\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

3. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

4. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials.

5. Given that $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

6. For which values of a and b, are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of $p(x)$ are not the zeroes of $q(x)$?