CHAPTER 2

POLYNOMIALS

(A) Main Concepts and Results

Meaning of a Polynomial

Degree of a polynomial

Coefficients

Monomials, Binomials etc.

Constant, Linear, Quadratic Polynomials etc.

Value of a polynomial for a given value of the variable

Zeroes of a polynomial

Remainder theorem

Factor theorem

Factorisation of a quadratic polynomial by splitting the middle term

Factorisation of algebraic expressions by using the Factor theorem

Algebraic identities –

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$x^2 - y^2 = (x + y)\ (x - y)$

$(x + a)\ (x + b) = x^2 + (a + b)\ x + ab$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3 + 3xy\ (x + y)$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3 = x^3 - y^3 - 3xy\ (x - y)$

$x^3 + y^3 = (x + y)\ (x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)\ (x^2 + xy + y^2)$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\ (x^2 + y^2 + z^2 - xy - yz - zx)$

(B) Multiple Choice Questions

Sample Question 1 : If $x^2 + kx + 6 = (x + 2)\ (x + 3)$ for all x, then the value of k is

(A) 1 (B) –1 (C) 5 (D) 3

Solution : Answer (C)

EXERCISE 2.1

Write the correct answer in each of the following :

1. Which one of the following is a polynomial?

(A) $\frac{x^2}{2} - \frac{2}{x^2}$ (B) $\sqrt{2x} - 1$

(C) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ (D) $\frac{x-1}{x+1}$

2. $\sqrt{2}$ is a polynomial of degree

(A) 2 (B) 0 (C) 1 (D) $\frac{1}{2}$

3. Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is

(A) 4 (B) 5 (C) 3 (D) 7

4. Degree of the zero polynomial is

(A) 0 (B) 1 (C) Any natural number

(D) Not defined

5. If $p(x) = x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to

(A) 0 (B) 1 (C) $4\sqrt{2}$ (D) $8\sqrt{2} + 1$

6. The value of the polynomial $5x - 4x^2 + 3$, when $x = -1$ is

(A) – 6 (B) 6 (C) 2 (D) –2

7. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

(A) 3 (B) $2x$ (C) 0 (D) 6

8. Zero of the zero polynomial is

(A) 0 (B) 1

(C) Any real number (D) Not defined

9. Zero of the polynomial $p(x) = 2x + 5$ is

(A) $-\frac{2}{5}$ (B) $-\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $\frac{5}{2}$

10. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is

(A) 2 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) –2

11. If $x^{51} + 51$ is divided by $x + 1$, the remainder is

(A) 0 (B) 1 (C) 49 (D) 50

12. If $x + 1$ is a factor of the polynomial $2x^2 + kx$, then the value of k is

(A) –3 (B) 4 (C) 2 (D) –2

13. $x + 1$ is a factor of the polynomial

(A) $x^3 + x^2 - x + 1$ (B) $x^3 + x^2 + x + 1$

(C) $x^4 + x^3 + x^2 + 1$ (D) $x^4 + 3x^3 + 3x^2 + x + 1$

14. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is

(A) $5 + x$ (B) $5 - x$ (C) $5x - 1$ (D) $10x$

15. The value of $249^2 - 248^2$ is

(A) 1^2 (B) 477 (C) 487 (D) 497

16. The factorisation of $4x^2 + 8x + 3$ is

(A) $(x + 1)(x + 3)$ (B) $(2x + 1)(2x + 3)$

(C) $(2x + 2)(2x + 5)$ (D) $(2x - 1)(2x - 3)$

17. Which of the following is a factor of $(x + y)^3 - (x^3 + y^3)$?

(A) $x^2 + y^2 + 2xy$ (B) $x^2 + y^2 - xy$ (C) xy^2 (D) $3xy$

18. The coefficient of x in the expansion of $(x + 3)^3$ is

(A) 1 (B) 9 (C) 18 (D) 27

19. If $\frac{x}{y} + \frac{y}{x} = -1$ $(x, y \neq 0)$, the value of $x^3 - y^3$ is

(A) 1 (B) –1 (C) 0 (D) $\frac{1}{2}$

20. If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then the value of b is

(A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

21. If $a + b + c = 0$, then $a^3 + b^3 + c^3$ is equal to

(A) 0 (B) abc (C) $3abc$ (D) $2abc$

(C) Short Answer Questions with Reasoning

Sample Question 1 : Write whether the following statements are **True** or **False**. Justify your answer.

(i) $\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$ is a polynomial (ii) $\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}}$ is a polynomial, $x \neq 0$

Solution :

(i) False, because the exponent of the variable is not a whole number.

(ii) True, because $\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}} = 6 + x$, which is a polynomial.

EXERCISE 2.2

1. Which of the following expressions are polynomials? Justify your answer:

(i) 8 (ii) $\sqrt{3}x^2 - 2x$ (iii) $1 - \sqrt{5x}$

(iv) $\frac{1}{5x^{-2}} + 5x + 7$ (v) $\frac{(x-2)(x-4)}{x}$ (vi) $\frac{1}{x+1}$

(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$ (viii) $\frac{1}{2x}$

2. Write whether the following statements are **True** or **False**. Justify your answer.

(i) A binomial can have atmost two terms

(ii) Every polynomial is a binomial

(iii) A binomial may have degree 5

(iv) Zero of a polynomial is always 0

(v) A polynomial cannot have more than one zero

(vi) The degree of the sum of two polynomials each of degree 5 is always 5.

(D) Short Answer Questions

Sample Question 1 :

(i) Check whether $p(x)$ is a multiple of $g(x)$ or not, where

$p(x) = x^3 - x + 1, \quad g(x) = 2 - 3x$

(ii) Check whether $g(x)$ is a factor of $p(x)$ or not, where

$$p(x) = 8x^3 - 6x^2 - 4x + 3, \quad g(x) = \frac{x}{3} - \frac{1}{4}$$

Solution :

(i) $p(x)$ will be a multiple of $g(x)$ if $g(x)$ divides $p(x)$.

Now, $g(x) = 2 - 3x = 0$ gives $x = \frac{2}{3}$

Remainder $= p\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right) + 1$

$$= \frac{8}{27} - \frac{2}{3} + 1 = \frac{17}{27}$$

Since remainder $\neq 0$, so, $p(x)$ is not a multiple of $g(x)$.

(ii) $g(x) = \frac{x}{3} - \frac{1}{4} = 0$ gives $x = \frac{3}{4}$

$g(x)$ will be a factor of $p(x)$ if $p\left(\frac{3}{4}\right) = 0$ (Factor theorem)

Now, $p\left(\frac{3}{4}\right) = 8\left(\frac{3}{4}\right)^3 - 6\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{4}\right) + 3$

$$= 8\times\frac{27}{64} - 6\times\frac{9}{16} - 3 + 3 = 0$$

Since, $p\left(\frac{3}{4}\right) = 0$, so, $g(x)$ is a factor of $p(x)$.

Sample Question 2 : Find the value of a, if $x - a$ is a factor of $x^3 - ax^2 + 2x + a - 1$.

Solution : Let $p(x) = x^3 - ax^2 + 2x + a - 1$

Since $x - a$ is a factor of $p(x)$, so $p(a) = 0$.

i.e., $a^3 - a(a)^2 + 2a + a - 1 = 0$

$a^3 - a^3 + 2a + a - 1 = 0$

$3a = 1$

Therefore, $a = \frac{1}{3}$

Sample Question 3 : (i) Without actually calculating the cubes, find the value of $48^3 - 30^3 - 18^3$.

(ii) Without finding the cubes, factorise $(x - y)^3 + (y - z)^3 + (z - x)^3$.

Solution : We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

If $x + y + z = 0$, then $x^3 + y^3 + z^3 - 3xyz = 0$ or $x^3 + y^3 + z^3 = 3xyz$.

(i) We have to find the value of $48^3 - 30^3 - 18^3 = 48^3 + (-30)^3 + (-18)^3$.

Here, $48 + (-30) + (-18) = 0$

So, $48^3 + (-30)^3 + (-18)^3 = 3 \times 48 \times (-30) \times (-18) = 77760$

(ii) Here, $(x - y) + (y - z) + (z - x) = 0$

Therefore, $(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$.

EXERCISE 2.3

1. Classify the following polynomials as polynomials in one variable, two variables etc.

(i) $x^2 + x + 1$ (ii) $y^3 - 5y$

(iii) $xy + yz + zx$ (iv) $x^2 - 2xy + y^2 + 1$

2. Determine the degree of each of the following polynomials :

(i) $2x - 1$ (ii) -10

(iii) $x^3 - 9x + 3x^5$ (iv) $y^3 (1 - y^4)$

3. For the polynomial

$\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$, write

(i) the degree of the polynomial

(ii) the coefficient of x^3

(iii) the coefficient of x^6

(iv) the constant term

4. Write the coefficient of x^2 in each of the following :

(i) $\frac{\pi}{6}x + x^2 - 1$ (ii) $3x - 5$

(iii) $(x - 1)(3x - 4)$ (iv) $(2x - 5)(2x^2 - 3x + 1)$

5. Classify the following as a constant, linear, quadratic and cubic polynomials :

(i) $2 - x^2 + x^3$ (ii) $3x^3$ (iii) $5t - \sqrt{7}$ (iv) $4 - 5y^2$

(v) 3 (vi) $2 + x$ (vii) $y^3 - y$ (viii) $1 + x + x^2$

(ix) t^2 (x) $\sqrt{2}x - 1$

6. Give an example of a polynomial, which is :

(i) monomial of degree 1

(ii) binomial of degree 20

(iii) trinomial of degree 2

7. Find the value of the polynomial $3x^3 - 4x^2 + 7x - 5$, when $x = 3$ and also when $x = -3$.

8. If $p(x) = x^2 - 4x + 3$, evaluate : $p(2) - p(-1) + p\left(\frac{1}{2}\right)$

9. Find $p(0), p(1), p(-2)$ for the following polynomials :

(i) $p(x) = 10x - 4x^2 - 3$ (ii) $p(y) = (y + 2)(y - 2)$

10. Verify whether the following are **True** or **False** :

(i) -3 is a zero of $x - 3$

(ii) $-\frac{1}{3}$ is a zero of $3x + 1$

(iii) $\frac{-4}{5}$ is a zero of $4 - 5y$

(iv) 0 and 2 are the zeroes of $t^2 - 2t$

(v) -3 is a zero of $y^2 + y - 6$

11. Find the zeroes of the polynomial in each of the following :

(i) $p(x) = x - 4$ (ii) $g(x) = 3 - 6x$

(iii) $q(x) = 2x - 7$ (iv) $h(y) = 2y$

12. Find the zeroes of the polynomial :

$p(x) = (x - 2)^2 - (x + 2)^2$

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial : $x^4 + 1; x - 1$

14. By Remainder Theorem find the remainder, when $p(x)$ is divided by $g(x)$, where

(i) $p(x) = x^3 - 2x^2 - 4x - 1, \quad g(x) = x + 1$

(ii) $p(x) = x^3 - 3x^2 + 4x + 50, \quad g(x) = x - 3$

(iii) $p(x) = 4x^3 - 12x^2 + 14x - 3, \quad g(x) = 2x - 1$

(iv) $p(x) = x^3 - 6x^2 + 2x - 4, \quad g(x) = 1 - \frac{3}{2}x$

15. Check whether $p(x)$ is a multiple of $g(x)$ or not :

(i) $p(x) = x^3 - 5x^2 + 4x - 3, \quad g(x) = x - 2$

(ii) $p(x) = 2x^3 - 11x^2 - 4x + 5, \quad g(x) = 2x + 1$

16. Show that :

(i) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$.

(ii) $2x - 3$ is a factor of $x + 2x^3 - 9x^2 + 12$.

17. Determine which of the following polynomials has $x - 2$ a factor :

(i) $3x^2 + 6x - 24$ (ii) $4x^2 + x - 2$

18. Show that $p - 1$ is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.

19. For what value of m is $x^3 - 2mx^2 + 16$ divisible by $x + 2$?

20. If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a.

21. Find the value of m so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.

22. If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a.

23. Factorise :

(i) $x^2 + 9x + 18$ (ii) $6x^2 + 7x - 3$

(iii) $2x^2 - 7x - 15$ (iv) $84 - 2r - 2r^2$

24. Factorise :

(i) $2x^3 - 3x^2 - 17x + 30$ (ii) $x^3 - 6x^2 + 11x - 6$

(iii) $x^3 + x^2 - 4x - 4$ (iv) $3x^3 - x^2 - 3x + 1$

25. Using suitable identity, evaluate the following:

(i) 103^3 (ii) 101×102 (iii) 999^2

26. Factorise the following:

(i) $4x^2 + 20x + 25$

(ii) $9y^2 - 66yz + 121z^2$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

27. Factorise the following :

(i) $9x^2 - 12x + 3$ (ii) $9x^2 - 12x + 4$

28. Expand the following :

(i) $(4a - b + 2c)^2$

(ii) $(3a - 5b - c)^2$

(iii) $(-x + 2y - 3z)^2$

29. Factorise the following :

(i) $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

(ii) $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$

(iii) $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24\,xz$

30. If $a + b + c = 9$ and $ab + bc + ca = 26$, find $a^2 + b^2 + c^2$.

31. Expand the following :

(i) $(3a - 2b)^3$ (ii) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$ (iii) $\left(4 - \frac{1}{3x}\right)^3$

32. Factorise the following :

(i) $1 - 64a^3 - 12a + 48a^2$

(ii) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

33. Find the following products :

(i) $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$ (ii) $(x^2 - 1)(x^4 + x^2 + 1)$

34. Factorise :

(i) $1 + 64x^3$ (ii) $a^3 - 2\sqrt{2}b^3$

35. Find the following product :

$(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$

36. Factorise :

(i) $a^3 - 8b^3 - 64c^3 - 24abc$ (ii) $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$.

37. Without actually calculating the cubes, find the value of :

(i) $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$ (ii) $(0.2)^3 - (0.3)^3 + (0.1)^3$

38. Without finding the cubes, factorise

$(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$

39. Find the value of

(i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$

(ii) $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$

40. Give possible expressions for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$.

(E) Long Answer Questions

Sample Question 1 : If $x + y = 12$ and $xy = 27$, find the value of $x^3 + y^3$.

Solution :

$$\begin{aligned} x^3 + y^3 &= (x + y)(x^2 - xy + y^2) \\ &= (x + y)[(x + y)^2 - 3xy] \\ &= 12[12^2 - 3 \times 27] \\ &= 12 \times 63 = 756 \end{aligned}$$

Alternative Solution :

$$\begin{aligned} x^3 + y^3 &= (x + y)^3 - 3xy\,(x + y) \\ &= 12^3 - 3 \times 27 \times 12 \\ &= 12\,[12^2 - 3 \times 27] \\ &= 12 \times 63 = 756 \end{aligned}$$

EXERCISE 2.4

1. If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

2. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a. Also find the remainder when $p(x)$ is divided by $x + 2$.

3. If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

4. Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$. [**Hint:** Factorise $x^2 - 3x + 2$]

5. Simplify $(2x - 5y)^3 - (2x + 5y)^3$.

6. Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$.

7. If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

8. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

9. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)\,(b + c)\,(c + a)$.