**Assignment-1 (NCERT Exercise 4.1- Value Of Determinant )**

1. If $A = \begin{bmatrix} -1 & -3 \\ 2 & 1 \end{bmatrix}$ find the determinant of $A^2 + 2A$
2. Evaluate the determinant $\Delta = \begin{vmatrix} 0 & \sin x & \cos x \\ -\sin x & 0 & \sin y \\ \cos x & -\sin y & 0 \end{vmatrix}$
3. Evaluate the following Determinant:
   i. $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$ where $i = \sqrt{-1}$
   ii. $\begin{vmatrix} sinx & -\sin y \\ cosx & cosy \end{vmatrix}$ where $x = 52^{o}$ , $y = 38^{o}$
4. Evaluate the determinant $\begin{vmatrix} log_4 9 & log_3 8 \\ log_4 3 & log_4 512 \end{vmatrix}$
5. Prove that the determinant $\begin{vmatrix} a & \sin x & \cos x \\ -\sin x & -a & 1 \\ \cos x & 1 & a \end{vmatrix}$ is independent of x
6. Evaluate the value of determinant $\Delta = \begin{vmatrix} 1 & \sin x & 1 \\ -\sin x & 1 & \sin x \\ -1 & -\sin x & 1 \end{vmatrix}$. Also show that $2 \le \Delta \le 4$
7. If $A(x_1, y_1)$ , $B(x_2, y_2)$ , and $C(x_3, y_3)$ are vertices of an equilateral triangle whose side is equal to a, then show that $\begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 3a^4$
8. If the points $(x_1, y_1)$ , $(x_2, y_2)$ , and $(x_3, y_3)$ are collinear, the show that then show that $x_1y_2 = x_2y_1$
9. Evaluate the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & 5 \\ 0 & 0 & -6 \end{vmatrix}$
10. Evaluate the determinant $\Delta = \begin{vmatrix} 0 & 6 & -8 \\ -6 & 0 & 4 \\ 8 & -4 & 0 \end{vmatrix}$

ANSWERS

1. 45
2. 0
3. i) $a^2 + b^2 + c^2 + d^2$ ii) 1
4. $\frac{15}{2}$
5. $-a^3$ which is independent of x
6. $\Delta = 2(1 + \sin^2 x)$
9. 24
10. 0

**Assignment-2** **(NCERT Exercise 4.2 –Properties Of Determinants )**

1. Without expanding prove that: $\begin{vmatrix} (x^a + x^{-a})^2 & (x^a - x^{-a})^2 & 1 \\ (x^b + x^{-b})^2 & (x^b - x^{-b})^2 & 1 \\ (x^c + x^{-c})^2 & (x^c - x^{-c})^2 & 1 \end{vmatrix}$ =0 where x, a, b >0
2. Using properties of determinants , prove that $\begin{vmatrix} b+c & a-b & a \\ a+c & b-c & b \\ b+a & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$
3. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1) = bc + ca + ab + abc$
4. find the value of f ( $\frac{\pi}{3}$ ) where f(θ) = $\begin{vmatrix} cos^{2\theta} & cos\theta sin\theta & -sin\theta \\ cos\theta sin\theta & sin^{2\theta} & cos\theta \\ sin\theta & -cos\theta & 0 \end{vmatrix}$
5. Show that $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ sin\beta & cos\beta & \cos(\beta+\delta) \\ sin\gamma & cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$
6. if A + B+ C =π , find the value of $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -sinB & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = 0$
7. if x, y, z are all different and Δ = $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$ = 0 then show that 1 + xyz = 0
8. Evaluate $\begin{vmatrix} (x+1)(x+2) & x+2 & 1 \\ (x+2)(x+3) & x+3 & 1 \\ (x+3)(x+4) & x+4 & 1 \end{vmatrix}$
9. Prove that $\begin{vmatrix} 2ab & a^2 & b^2 \\ a^2 & b^2 & 2ab \\ b^2 & 2ab & a^2 \end{vmatrix}$ = - $(a^3 + b^3)^2$
10. Prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ = $(ab + bc + ca)^3$
11. if a,b, c are all positive and $p^{th}$, $q^{th}$ , $r^{th}$ term of a G.P , then prove that $\begin{vmatrix} loga & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ = 0
12. Prove that $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$
13. Show that: $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$
14. show that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (c+a)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$
15. if ( a+ b+ c ) = 2s , prove that $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}$ = $2s^3$( s-a ) (s – b) (s –c )

16. Show that $\begin{vmatrix} a & b-c & b+c \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

17. Prove that $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$

18. Prove that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & cb \\ ca & cb & b^2+a^2 \end{vmatrix} = 4a^2b^2c^2$

19. Show that $\begin{vmatrix} 1 & log_x y & log_x z \\ log_y x & 1 & log_y z \\ log_z x & log_z y & 1 \end{vmatrix} = 0$

20. if $\Delta = \begin{vmatrix} \frac{1}{z} & \frac{1}{z} & -\frac{(x+y)}{z^2} \\ \frac{-(y+z)}{x^2} & \frac{1}{x} & \frac{1}{x} \\ \frac{-y(y+z)}{x^2 z} & \frac{(x+z)}{xz} & \frac{-y(x-y)}{xz^2} \end{vmatrix}$, then show that $\Delta = 0$

21. Solve the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

22. Show that x = 2 is a root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & -2x & x+2 \end{vmatrix} = 0$ and solve it completely

23. Solve the following determinant equation $\begin{vmatrix} 3+x & 3-x & 3-x \\ 3-x & 3+x & 3-x \\ 3-x & 3-x & 3+x \end{vmatrix} = 0$

24. If a + b + c = 0 and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ ,then show that x = 0 , or x = $\pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

25. Show that $\Delta = \begin{vmatrix} -2a & a+b & a+c \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$

ANSWERS

1. 0
4. 1
8. -2
21. x = $\frac{11}{3}, \frac{11}{3}, \frac{2}{3}$
22. x = 0, 0 and x= 9

**Assignments-3 ( NCERT Exercise 4.3 –Adjoint Of Square Matrix, Properties Of Inverse Of A Matrix)**

1. if A= $\begin{bmatrix} 1 & tanx \\ -tanx & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} cos2x & -\sin 2x \\ sin2x & \cos 2x \end{bmatrix}$
2. For the matrix A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y so that $A^2 + xI - yA = 0$ . Hence , find $A^{-1}$
3. If A= $\begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$, show that $A^{-1} = A^2$
4. Find a 2 X 2 matrix B such that B $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$
5. Find the matrix X for which $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$
6. Find the matrix A satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
7. Find the matrix A = $\begin{bmatrix} 2 & 4 \\ 1 & -3 \end{bmatrix}$, verify that $(A^{-1})^T = (A^T)^{-1}$

ANSWERS

2. $A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$
4. B = $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$
5. X = $\begin{bmatrix} 6 & 2 \\ 11/2 & 2 \end{bmatrix}$
6. A = $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

**Assignments- 4** **( NCERT Exercise 4.4 Solution Of A System Of Linear Equation)**

1. Find $A^{-1}$ where $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$.Hence solve the system of equations:$-x + 2y + 5z = 2, 2x - 3y + z = 15, -x + y + z = -3$
2. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find $AB$ and hence solve the system of equations: $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$.
3. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$. Find $A^{-1}$ and hence solve the system of equations: $x + 2y + z = 4, -x + y + z = 0, x - 3y + z = 2$
4. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x + y + 2z = 1, -x - 2y + z = 4, x - 2y + 3z = 0$

ANSWERS

1. $A^{-1} = \frac{1}{-7}\begin{bmatrix} -4 & 3 & 17 \\ -3 & 4 & 11 \\ -1 & -1 & -1 \end{bmatrix}$
   X= 2, y = -3 , z = 2 is required solution
2. AB= $6I \Rightarrow A^{-1} = \frac{1}{6}B$
   x = 2 , y = -1, z = 4 is required solution
3. $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$
   x = $\frac{9}{5}$, y = $\frac{2}{5}$, **z = =** $\frac{7}{5}$ is required solution
4. $A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$
   x = -4 , y = 1 , z = 2