Assignment-1 (NCERT Exercise 2.1- Inverse of sine ,cosine, tangent ,secant function)

1. Find the principal value of the following:
   i. $\text{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$
   ii. $\tan^{-1}(-1)$
   iii. $\sec^{-1}(-2)$

2. Find the value of the following (using principal value)
   i. $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
   ii. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$
   iii. $\sec^{-1}(\sqrt{2}) + \text{cosec}^{-1}(\sqrt{2})$
   iv. $\tan^{-1}(1)$ - $\cot^{-1}(-1)$
   v. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
   vi. $\tan^{-1}(1) + \cot^{-1}(1) + \sin^{-1}(-1)$

3. i. If $\tan^{-1}\left(\frac{3}{4}\right)$= x , then find the value of cos x

   ii. If $\cot^{-1}\left(-\frac{1}{5}\right)$= x , then find the value of sin x

4. Evaluate $\sec^2(\tan^{-1} 2)$

ANSWERS

1.
   i. $\frac{\pi}{3}, \frac{\pi}{3} \epsilon \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
   ii. $-\frac{\pi}{4}, -\frac{\pi}{4} \epsilon \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
   iii. $\frac{2\pi}{3}, \frac{2\pi}{3} \epsilon [0, \pi] - \{\frac{\pi}{3}\}$

2.
   i. $-\frac{\pi}{3}$
   ii. 0
   iii. $\frac{\pi}{2}$
   iv. $\frac{-\pi}{2}$
   v. $\frac{\pi}{6}$
   vi. π

3.
   i. $\frac{4}{5}$
   ii. $\frac{5}{\sqrt{26}}$

4. 5

Assignment-2 **(NCERT Exercise 2.2 –Properties Of Inverse Trigonometric Functions)**

1. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$, then write the value of $x + y + xy$.
2. If $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}x\right) = 1$, then find the value of x
3. Simplify: $\sin^{-1}\left(\frac{5}{13}cosx + \frac{12}{13}\sin x\right)$.
4. Prove that : $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$
5. Prove that : $\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$
6. Prove that : $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$
7. Prove that: $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$
8. Prove that: $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$
9. Prove that $tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$.
10. Prove that $\tan^{-1}\left(\frac{3\sin\theta}{5+3\cos 2\theta}\right) + \tan^{-1}\left(\frac{1}{4}\tan\theta\right) = \theta$, where $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$
11. Prove that $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$
12. Prove that $4(\cot^{-1}3 + \text{cosec}^{-1}\sqrt{5}) = \pi$
13. Prove that $\sin[\cot^{-1}\{\cos(\tan^{-1}1)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$
14. Express each of the following in the simplest form.
    i. $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-\pi}{2} < x < \frac{\pi}{2}$
    ii. $\tan^{-1}\sqrt{\frac{1-cos\,x}{1+cos\,x}}$, $-\pi < x < \pi$
    iii. $\tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right)$
    iv. $\sin^{-1}\sqrt{\frac{x}{1+x}}$
15. Solve the following equations for x :
    i. $\sin^{-1}\frac{3x}{5} + \sin^{-1}\frac{4x}{5} = \sin^{-1}x$
    ii. $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$
    iii. $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$
16. if $y = \cot^{-1}(\sqrt{cosx}) - \tan^{-1}(\sqrt{cosx})$, *prove that* $\sin y = \tan^2\left(\frac{x}{2}\right)$
17. Prove that : $\cos^{-1}\left(\frac{cos\alpha + cos\beta}{1 + cos\alpha\ cos\beta}\right) = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$
18. if $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right] = \alpha$ then prove that $x^2 = \sin 2\alpha$
19. Solve the equation $\tan^{-1}(\sqrt{x + x^2}) + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$
20. Prove that : $\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$
21. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$, prove that a + b + c = abc
22. if $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$
23. if a > b > c >0 , prove that $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right) = \pi$
24. Prove that : $\tan^{-1}\frac{yz}{xr} + \tan^{-1}\frac{zx}{yr} + \tan^{-1}\frac{xy}{zr} = \frac{\pi}{2}$, *where* $x^2 + y^2 + z^2 = r^2$

25. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$ prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = sin^2\alpha$.
26. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$
27. Prove that $\tan^{-1}\left(\frac{1-x}{1+x}\right) - \tan^{-1}\left(\frac{1-y}{1+y}\right) = \sin^{-1}\left(\frac{y-x}{\sqrt{1+x^2}\sqrt{1+y^2}}\right)$

ANSWERS

1. 1
2. $\frac{1}{2}$
3. $\tan^{-1}\frac{5}{12} + x$
14.
   i. $\frac{\pi}{4} + \frac{x}{2}$
   ii. $\frac{x}{2}\ if\ 0 < x < \pi$ and $\frac{x}{2}\ if - \pi < x < 0$
   iii. $2\ \tan^{-1}\sqrt{x}$
   iv. $\tan^{-1}\sqrt{\frac{x}{a}}$
15.
   i. $x = \pm 1$
   ii. $2 - \sqrt{3}$
   iii. $x = 13$
19. $x = 0, -1$

Assignments- 3 **(NCERT Exercise 2.3 Solution Of A System Of Linear Equation)**

1. Find A^{-1} where $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$.Hence solve the system of equations:$-x + 2y + 5z = 2, 2x - 3y + z = 15, -x + y + z = -3$
2. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and$B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of equations: $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$.
3. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$. Find A^{-1} and hence solve the system of equations: $x + 2y + z = 4, -x + y + z = 0, x - 3y + z = 2$
4. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x + y + 2z = 1, -x - 2y + z = 4, x - 2y + 3z = 0$

ANSWERS

1. $A^{-1} = \frac{1}{-7}\begin{bmatrix} -4 & 3 & 17 \\ -3 & 4 & 11 \\ -1 & -1 & -1 \end{bmatrix}$
   X= 2, y = -3 , z = 2 is required solution
2. AB= $6I \Rightarrow A^{-1} = \frac{1}{6}B$
   x = 2 , y = -1, z = 4 is required solution
3. $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$
   x = $\frac{9}{5}$, y = $\frac{2}{5}$, **z = =** $\frac{7}{5}$ is required solution
4. $A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$
   x = -4 , y = 1 , z = 2