Assignment-1 (NCERT Exercise 1.1- Types of Relations)

1. Let N be the set of natural numbers and a relation R be is defined over N as: $R = \{ (x, y) : x, y \in N, x + 2y = 10\}$. Check it for reflexivity, symmetricity and transitivity.
2. Prove that the relation R on the set Z of all integers defined by $R = \{(a, b) : a, b \in Z \text{ and } (a - b)$ is divisible 5. Prove that R is an equivalence Relation.
3. Let N be the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\ R(c, d)$ if $ad = bc$ *for all* $(a, b), (c, d) \in N X N$. Prove that R is an equivalence relation.
4. Let R be a relation defined on Z such that $R = \{ (a, b): a, b \in Z$, and $|a - b| \leq 5\}$
5. Let a relation R_1 on the set R of real numbers be defined as $(a,b) \in R_1 \Leftrightarrow 1 + ab > 0$ for all $a,b \in R$ .show that R_1 is reflexive and symmetric but not transitive.
6. A relation R is defined by $R = \{(a, b) : a, b \in N$ and b is divisible by a$\}$. Check it for reflexivity, symmetricity and transitivity.
7. Let N be the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\ R(c, d)$ if $ad(b + c) = bc(a + d)$. Check whether R is an equivalence relation.
8. Prove that a relation R on a set A is symmetric iff $R = R^{-1}$
9. If R is an equivalence relation on a set A , then R^{-1} is also an equivalence relation on A

ANSWERS

1. R is nor reflexive, symmetric and transitive
2. R is an equivalence relation.
3. R is an equivalence relation.
4. R is reflexive, symmetric but not transitive.
5. R_1 is reflexive , symmetric but not transitive
6. R is reflexive, transitive but not symmetric.
7. R is an equivalence relation.
9. R is an equivalence relation on A

Assignment-2 (NCERT Exercise 1.2- Functions)

1. Let A = { x ϵ R : -1 ≤ x ≤ 1} show that f : A→ B given by f(x) = x$|x|$ is a one – one and onto function.
2. Show that the function f : R →R , given by f(x) = ax + b , where a , b ϵ R, a ≠ 0 is a bijection.
3. Show that the function f : R→ R , given by f(x) = sinx is neither one –one nor onto
4. Let f : N-{1}→ N be defined by f(n) = the highest prime factor of n. Show that f is neither one-one nor onto. Find the range of f
5. Let $f \rightarrow W \times W$, be defined as $f(x) = \begin{cases} x+1, if \ x \ is \ even \\ x-1, \ if \ x \ is \ odd \end{cases}$ where x is the set of whole numbers. Show that f is a bijection
6. Show that the identity function I_N : N→N defined by I_N (x) = x for all x ϵ N is an onto function but $I_N + I_N$: N→ N defined as ($I_N + I_N$) (x) = I_N (x) I_N + I_N (x) is not onto
7. Prove that the function f : N→N defined by f(x) = x^2 + x + 1 is one –one but not onto.
8. Show that the function f : R – { b} →R – {1} given by f(x) = $\frac{x-a}{x-b}$ is a bijection, where a ≠b

ANSWERS

1. Range of f is the set of all prime numbers.

Assignment-3 (NCERT Exercise 1.3- Composite Of Functions , Invertible Functions

1. Let $f : R \to R$; $f(x) = \sin x$ and : $R \to R$; $g(x) = x^2$, find fog and gof . Also show that fog ≠ gof
2. If f : $R \to R$ is given by $f(x) = (3 - x^3)^{1/3}$ then show that fof = I_R , where I_R is the identity function on R.
3. Let R be the set of all real numbers. If f : $R \to R$ is given by f(x) = 3x + 2, $\forall$ x ϵ R and g : R→R is given by g(x)= $\frac{x}{x^2+1}$ $\forall$ x ϵ R then find i) gof ii) fog iii) fof iv) gog
4. Let f(x) = x^2 and g(x) = $\sqrt{x}$, both f and g are defined from R to R then show that gof(-4) = 4.
5. If f : $R \to R$ and g : $R \to R$ be function defined by f(x) = [x] and g(x) = $|x|$ then evaluate the following
   i) (f + 2g)(-2) ii) (f – g)$\left(\frac{1}{2}\right)$ iii) gof $\left(\frac{7}{3}\right)$ - fog$\left(\frac{7}{3}\right)$
6. Let f : $R \to R$ be signum function defined as f(x) =$\begin{cases} 1 \ , if\ x > 0 \\ 0, if\ \ x = 0 \\ -1\ , if\ x < 0 \end{cases}$ and
   g: $R \to R$, be the greatest integer function defined as g(x) $=$ [x], then find whethe gof and fog coincide in (0, 1)
7. If f(x) = e^x and g(x) = log_{e^x} (x > 0), find fog and gof. Are they equal?
8. If f(x) = $\frac{1}{x}$ and g(x) = 0 are two real functions, show that fog is not defined.
9. If f(x) = $\frac{x-1}{x+1}$, x ≠ -1 then show that fof (x) = - $\frac{1}{x}$, x ≠ 0, -1
10. If A = { 1,2,3,4,5} , B = {2,4,6,8,10 } and f : A→B is defined by f(x)= 2x, find f and f^{-1} as a set of ordered pairs
11. Let f : $R \to R$ *be defined by* f(x) = 5x - 9. Show that f is invertible and hence find f^{-1}
12. Show that f : R-{0}→ R – {0} given by f(x) = $\frac{2}{x}$ is invertible and it is inverse of itself.
13. Let a function f : $R \to R$ *be defined by* $f(x) = 1 + \alpha x , \alpha \neq 0, for\ all\ x \in$ R . show that f is invertible and find its inverse function. Also find the value(s) of α for which inverse of f is itself
14. Let f : N U {0}→ N U {0} be defined by $f(x) = \begin{cases} x + 1, if\ \ x\ is\ \ odd \\ \ \ x - 1,\ if\ x\ is\ even \end{cases}$. Show that f is invertible and f =f^{-1}
15. Let f : N→R be a function defined by f(x) = $4x^2$ + 12x + 15. Show that f : N→ Range (f) is invertible. Find the inverse of f

ANSWERS

1. gof = $\sin^2 x$, fog = $\sin x^2$
3. i) $\frac{3x+2}{9x^2+12x+5}$ ii) $\frac{3x+2x^2+2}{x^2+1}$ iii) 9x+ 8 iv) $\frac{x(x^2+1)}{x^4+3x^2+1}$
5. i) 2 ii) -$\frac{1}{2}$ iii) 0
7. fog = x , gof = x but domain of fog and gof are not equal so gof ≠ fog
10. f = {(1,2),(2,4),(3,6),(4,8),(5,10) , f^{-1} = {(1,2),(4,2),(6,3),(8,4),(10,5)}
11. f^{-1} = $\frac{x+9}{5}$
13. f^{-1} = $\frac{x-\alpha}{\alpha}$, α = -1
15. f^{-1} = $\frac{-3+\sqrt{x-6}}{2}$

Assignment-4 (NCERT Exercise 1.4- Binary Operations)

1. let S ={ $a+\sqrt{2}\,b$: a , b ϵ Z } Then Prove that an operation on S defined by $(a_1+\sqrt{2}\,b_1)$ $(a_2+\sqrt{2}\,b_2) = (a_1+a_2) + \sqrt{2}\,(b_1+b_2)$ for all a_1, b_1, a_2, b_2 ϵ Z is a binary operation on S
2. Show that the operation ˅ and ^ on R defined as
   a V b = maximum of a and b
   a ^ b = minimum of a and b are binary operation on R
3. On the set Q of all rational numbers an operation is defined by a * b = 1+ ab. Show that * is a binary operation on Q.
4. Let M be the set of all singular matrices of the form $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$, where x is a non – zero real number. On M , Let * be an operation defined by A* B = AB for all A, B ϵ M . prove that * is a binary operation
5. Let * be a binary operation on R the set of all real numbers, defined by a* b =$\sqrt{a^2+b^2}$ for all a , b ϵ R. Show that * is commutative
6. If the operation * is defined on the set Q of all rational numbers by the rule a * b = $\frac{ab}{3}$ for all a , b ϵ Q
7. Discuss the commutativity and associativity of binary operation ∗ defined on Q by the rule $a * b = a - b + ab \text{ for all } a, b \in A$.
8. Let A be a non-empty set and S be the set of all functions from A to itself. Show that the composition of functions 'O' is a non- commutative binary operations on S. Also, prove that 'O' is an associative binary operation on S.
9. Let A = N X N and * be a binary operation on A defined by (a, b) *(c, d) = (ac , bd) for all a, b, c, d ϵ N Show that '*' is commutative and associative binary operation on A
10. Let S be a non-empty set and P(S) be the power set of set S. Find the identity element for the union (U) as a binary operation on P(S).Also find the identity element for intersection (∩) as a binary operation on P(S)
11. Let * be a binary operation on set Q –{1} defined by a* b= a+ b- ab ; a , b ϵ Q –{1}. find the identity element With respect to * on Q . prove that every element Q-{1} is invertible.
12. On the set R-{-1} a binary operation * is defined by a * b = a + b+ ab for all a, b ϵ R –{-1}. Prove that * is commutative as well as associative on R- {-1} . Find the identity element (if exist).
13. On the set S = M(x) = $\left\{\begin{bmatrix} x & x \\ x & x \end{bmatrix} : x \in R\right\}$ of 2 X 2 matrices, find the identity element for the multiplication of matrices as a binary operation.
14. Let A = N X N and * be a binary operation on A defined by (a, b) * (c,d) = (a+c, b + d). Show that * is commutative and associative. Find the identity element for * on A , if any.
15. If A = N X N and '*' on A is defined by (a, b) * (c , d) =(ad + bc, bd) for all (a, b) ,(c,d)ϵA , then show that
    i. '*' is a binary operation on A
    ii. '*' is commutative on A
    iii. '*' is associative on A.
    iv. A has no identity element with respect to the given operation.
16. If A = Q X Q and '*' on A is defined by (a, b)*(c,d)= (ac, b+ ad) for all (a,b),(c,d) ϵ A , then
    i. Show that '*' is a binary operation on A.
    ii. Show that the operation '*' is non- commutative on A
    iii. Show that operation '*' is associative on A
    iv. Find the identity element in A
    v. Find the invertible elements of A

ANSWERS

1. S is a binary operation
7. * is neither commutative nor associative
10. Φ is the identity element in P(S) for the operation 'U' and
    S is the identity element for intersection $(\cap)$ on P(S)
12. 0 is the identity element
13. $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ is the identity element
14. There does not exist any identity element for * on A
16. Iv) (1,0) is the identity element in A v) $(1/a \quad -b/a)$