Matrix

VERY SHORT QUESTION (1 MARKS)

1. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3. (AI 2016)
2. Write the element a_{23} of a 3×3 matrix A=$|a_{ij}|$ whose element a_{ij} are given by $a_{ij} = \left|\frac{i-j}{2}\right|$ (Delhi 2015)
3. The elements a_{ij} of a 3×3 matrix are given by$a_{ij} = \frac{1}{2}|-3i + j|$. Write the value of element a_{32} (AI 2014C)
4. For a 2×2 matrix A=$[a_{ij}]$, whose elements are given by$a_{ij} = \frac{(i+2j)^2}{4}$, write the value of a_{21}. (Delhi2012C)
5. For a 2×2 matrix A=(a_{ij}) , whose elements are given by$a_{ij} = \frac{i}{j}$, write the value of a_{21}. (Delhi2011)
6. If a matrix has 5 elements, then write all possible orders it can have. (AI 2011)

Types of Equation

VERY SHORT QUESTION (1 MARKS)

1. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$. (AI 2014)
2. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+b & b+2 \\ 8 & a-8b \end{bmatrix}$, write the value of $a - 2b$. (Foreign 2014)
3. If $\begin{bmatrix} x.y & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, write the value of $(x + y + z)$. (Delhi 2014C)
4. Find the value of a if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ (Delhi 2013)
5. Find the value of b if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ (Delhi 2013)
6. If $\begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$, write the value of $(x + y + z)$. (AI 2013C)
7. If $\begin{bmatrix} 2x+1 & 2y \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix}$, write the value of $(x + y)$. (AI 2012C)
8. If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, find the value of y. (Delhi 2011C)
9. If $A = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$, then for what the value of α is A an identity matrix. (Delhi 2010)
10. If $\begin{bmatrix} x+y & 1 \\ 2y & 5 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix}$, then find x. (Delhi 2010C)
11. If $\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$, then find y. (AI 2010C)
12. Find the value of x and y, if $\begin{bmatrix} 3x+y & -y \\ 2y-3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$. (AI 2009)
13. Find the value of x from the following:$\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$. (AI 2009)
14. If $\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$, find the value of x. (Delhi 2009 C)
15. If $\begin{bmatrix} 2x & 1 \\ 5 & x+2y \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 5 & 0 \end{bmatrix}$, find the values of x and y. (AI 2009 C)
16. If $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the values of x and y. (Delhi 2008 C)
17. If $\begin{bmatrix} x+2y & 3y \\ 4x & 2 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 8 & 2 \end{bmatrix}$, find the values of x and y. (AI 2008 C)

Operations On Matrices

VERY SHORT QUESTION (1 MARKS)

1. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A-I)^3 + (A+I)^3 - 7A$. (Delhi 2016)
2. If $\begin{bmatrix}2 & 1 & 3\end{bmatrix}\begin{bmatrix}-1 & 0 & -1\\ -1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\\ -1\end{bmatrix} = A$, then write the order of matrix A. (Foreign 2016)
3. Solve the following matrix equation for x: $\begin{bmatrix}x & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -2 & 0\end{bmatrix} = O$ (Delhi 2014)
4. If $2\begin{bmatrix}3 & 4\\ 5 & x\end{bmatrix} + \begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\ 10 & 5\end{bmatrix}$, find $(x-y)$. (Delhi 2014)
5. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (A+I)^3$, *where* I is an identity matrix (AI 2014)
6. If $\begin{pmatrix}2x & 4\end{pmatrix}\begin{pmatrix}x\\ -8\end{pmatrix} = 0$, find the positive value of x. (AI 2014C)
7. If $\begin{bmatrix}9 & -1 & 4\\ -2 & 1 & 3\end{bmatrix} = A + \begin{bmatrix}1 & 2 & -1\\ 0 & 4 & 9\end{bmatrix}$, then find the matrix A. (Delhi 2013)
8. If matrix $A = \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ and $A^2 = kA$, then write the value of k. (AI 2013)
9. If matrix $A = \begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$ and $A^2 = pA$, then write the value of p. (AI 2013)
10. If matrix $A = \begin{bmatrix}3 & -3\\ -3 & 3\end{bmatrix}$ and $A^2 = \lambda A$, then write the value of λ. (AI 2013)
11. If $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$, then write the value of $(x+y)$. (Delhi 2013C, AI 2012)
12. Simplify: $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$ (Delhi 2012)
13. If $\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\ -2 & 4\end{bmatrix} = \begin{bmatrix}-4 & 6\\ -9 & x\end{bmatrix}$, write the value of x. (Delhi 2012)
14. Find the value of $x+y$ from the following equation: $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix} + \begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix} = \begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$ (AI 2012)
15. If $3A - B = \begin{bmatrix}5 & 0\\ 1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$ then find the matrix A. (Delhi 2012C)
16. If $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 5\end{bmatrix} = \begin{bmatrix}7 & 11\\ k & 23\end{bmatrix}$, then write the value of k. (Delhi 2010)
17. If A is a matrix of order 3×4 and B is a matrix of order 4×3, find the order of (AB). (Delhi 2010C)
18. Find the value of x *and* y, if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$ (Delhi 2008)
19. If $A = \begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}$ *and* $B = \begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$ then find $A - B$. (AI 2008C)

LONG QUESTION "1" (4 MARKS)

1. Let $A = \begin{pmatrix}2 & -1\\ 3 & 4\end{pmatrix}$, $B = \begin{pmatrix}5 & 2\\ 7 & 4\end{pmatrix}$, $C = \begin{pmatrix}2 & 5\\ 3 & 8\end{pmatrix}$, *find a matrix* D *such that* CD – AB = O. (Delhi 2017)
2. Find matrix A such that $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A = \begin{bmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{bmatrix}$ (AI 2017)
3. If $A = \begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, find $A^2 - 5A + 4I$ and find a matrix X such that $A^2 - 5A + 4I + X = O$ (Delhi 2015)
4. Three schools A, B *and* C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of $Rs\ 25$, $Rs\ 100$ *and* $Rs\ 50$ each. The numbers of articles sold are given below

Article/school	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also, find the total funds collected for the purpose. Write one value generated by the above situation. (Delhi 2015)

5. To promote the making of toilets for women, an organisation tried to generate awareness through (i) house calls (ii) letters (iii) announcements. The cost for each mode per attempt is given below :
(i) Rs 50 (ii) Rs 20 (iii) Rs 40

The number of attempts made in three villages X , Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
$\boldsymbol{Y}$	**300**	250	75
Z	500	400	150

Find the total cost incurred by the organisation for the three villages separately, using matrices .Write one value generated by the organisation in the society. (AI 2015)

6. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then find the values of a and b. (Foreign 2015)

7. In a parliament election, a political party hired a public relations firm to promote its candidates in three ways – telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{matrix} Telephone \\ House\ call \\ Letters \end{matrix}$$

The number of contacts of each type made in two cities X and Y is given in matrix B as

$$B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{matrix} \text{City X} \\ \text{City Y} \end{matrix}$$

Find the total amount spent by the party in the two cities .What should one consider before casting his /her vote party promotional activity or their social activities? (Foreign 2015)

8. If $[2x \quad 3] \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = O$, find x. (Delhi 2015C)

9. A trust fund, Rs 35,000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bond pay 10% interest per annum which will be given to an N.G.O (Cancer Aid Society). Using matrix multiplication, determine how to divide Rs35, 000 among two types of bonds if trust fund obtains an annual total interest of Rs 3,200. What are the values reflected in this question?

(AI 2015C)

10. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ then find the value of $A^2 - 3A + 2I$. (AI 2010)

11. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then verify that $A^2 - 4A - 5I = O$. (Delhi 2008)

12. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 5A + 7I = O$. (Delhi 2007C)

Transpose Of A Matrix

VERY SHORT QUESTION (1 MARKS)

1. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$. (AI 2012)
2. If $\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}'$, then find a. (Delhi 2010C)
3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $A + A'$. (AI 2010C)
4. If $\begin{pmatrix} 2x+y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 6 & 4 \end{pmatrix}'$, then find x. (AI 2010C)
5. If matrix $A = [1 \quad 2 \quad 3]$, then find AA' where A' is the transpose of matrix A. (Delhi 2009)
6. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ then find $A + A'$ where $A'$the transpose of matrix A. (Delhi 2009C)

LONG QUESTION "1" (4 MARKS)

1. For the following matrices A and B, verify that $(AB)'=B'A'$. $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = [-1 \quad 2 \quad 1]$ (AI 2010)

SYMMETRIC AND SKEW SYMMETRIC MATRICES

VERY SHORT QUESTION (1 MARKS)

1. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & 1 \end{bmatrix}$ is given to be symmetric, find values of a and b. (Delhi 2016)
2. If $A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$ is written as $A = P + Q$ where P is symmetric matrix and Q is a skew symmetric matrix, then write the matrix P (Foreign 2016)
3. Express the Matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (AI 2015C)
4. Write a Matrix 2×2 which is both symmetric and skew symmetric. (Delhi 2014C)
5. For what value of Matrix x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$, a skew symmetric matrix? (AI 2013)

SHORT QUESTION (2 MARKS)

1. Show that all the diagonal elements of a skew symmetric matrix are zero. (Delhi 2017)
2. If A is a skew symmetric matrix of order 3, then prove that $\det A = 0$. (AI 2017)

LONG QUESTION "1" (4 MARKS)

1. Express the Matrix $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ 1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix and verify your result (AI 2015C)

2. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$, express A as a sum of two matrices such that one is symmetric and the other is skew symmetric. (Delhi 2008)

Elementary Operation (Transformation) Of Matrices

VERY SHORT QUESTION (1 MARKS)

1. Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix operation: $\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$ (AI 2016)
2. Use elementary column operation $C_2 \to C_2 - 2C_1$ in the following matrix operation:

$$\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$$ (Foreign 2014)

Invertible Matrices

LONG QUESTION"1" (4 MARKS)

1. Using elementary row transformations, find the inverse of the following matrix. $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (Delhi 2010)

LONG QUESTION"2" (6 MARKS)

1. Using elementary operations , find the inverse of the following matrix: $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ (Delhi 2012)

2. Using elementary operations , find the inverse of the following matrix: $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$ (Delhi 2011)

3. Find the inverse of the following matrix using elementary operations: $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ (AI 2010)

4. Obtain the inverse of the following matrix using elementary operations: $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ (AI 2009)

5. Using elementary operations , find the inverse of the following matrix: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (AI 2008)