Assignment-1 (NCERT Exercise 9.2- Formation of a Differential Equation)

1. Show that $y = be^x + ce^{2x}$ is the solution of the differential equation, $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.
2. Show that the function $y = A \cos x + B \sin x$ is a solution of differential equation $\frac{d^2y}{dx^2} + y = 0$.
3. Show that $y = \frac{C-x}{1+Cx}$ is a solution of the differential equation $(1 + x^2)\frac{dy}{dx} + ((1 + y^2) = 0$.
4. Verify that $y = \log(x + \sqrt{x^2 + a^2})$ is a solution of the differential equation $(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.
5. Show that $y = e^x$ (A cos x + B Sin x) is the solution of differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.
6. Show that $y = Ax + \frac{B}{x}$ is a solution of the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.
7. Show that $y = Cx + \frac{a}{C}$ is a solution of the differential equation $y = x\frac{dy}{dx} + \frac{a}{(\frac{dy}{dx})}$.

Assignment-2 **(NCERT Exercise 9.3 –Solution Of Differential Equations)**

1. Form the differential equation of the family of curves y = A sin 2x + B cos 2x.
2. Find the differential equation of the family of curves y =a $(x - a)^2$
   Where a is an arbitrary constant.
3. Find the differential equation of the circle of radius r and centre as (a, b).
4. Form the differential equation of the family of curves y = $(\sin^{-1} x)^2$ + A $\cos^{-1} x$ + B, Where A and B are arbitrary constants.
5. Show that the differential equation that represents all parabolas having their axis of symmetry coincident with the axis of y is x $\frac{d^2y}{dx^2}$ - $\frac{dy}{dx}$ = 0.
6. Find the differential equation of all circles passing through the origin and having its centre on the y – axis.

ANSWERS

1. $\frac{d^2y}{dx^2} + 4y = 0$
2. $(\frac{dy}{dx})^3 - 4xy\left(\frac{dy}{dx}\right) + 8y^2 = 0$
3. $[1+(\frac{dy}{dx})^2]^3 = r^2(\frac{d^2y}{dx^2})^2$
4. $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$
5. $x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0.$
6. $(x^2-y^2)\frac{dy}{dx} - 2xy = 0$

Assignments-3 (NCERT Exercise 9.4 –Solution of Differential Equation of the First Order and First Degree)

1. Solve the differential equation $\frac{dy}{dx} = x^5 \tan^{-1}(x^3)$.
2. Solve the differential equation $\cos x \frac{dy}{dx} = \cos 3x - \cos 2x$.
3. $\frac{dy}{dx} + \frac{1+y^2}{2y} = 0$.
4. Solve the following differential equations:
   a. $\frac{dy}{dx} = e^{x+y} + x^2 e^y$
   b. $\frac{dy}{dx} = e^{x-y} + e^{2\log x - y}$
   c. $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$.
5. Solve the following differential equations:
   a. $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y + y\cos Y}$
   b. $\left(y - x\frac{dy}{dx}\right) = y$
   c. $3e^x \tan y\, dx + (1 - e^x)\, Sec^2\, y\, dy = 0$
   d. $(x^2 - yx^2)\, dy + (y^2 + xy^2)\, dx = 0$
   e. $(x - y^2 x)\, dx - (y - x^2 y)\, dy = 0$
6. Solve the following differential equations:
   a. $\frac{dy}{dx} = xy + x + y + 1$
   b. $x(1 + y^2)\, dx + y(1 + x^2)\, dy = 0$
7. Solve the following differential equations:
   a. $\frac{dy}{dx} = \frac{xy+y}{xy+x}$
   b. $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$
   c. $x\cos^2 y\, dx = y\cos^2 x\, dy$
   d. $\tan y\left(\frac{dy}{dx}\right) = \sin(x + y) + \sin(x - y)$
8. Solve the following differential equations:
   a. $\tan y \frac{dy}{dx} = \sin(x + y) + \sin(x - y)$
   b. $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$
   c. $y - x\frac{dy}{dx} = 3\left(1 + x^2 \frac{dy}{dx}\right)$
   d. $y\sqrt{1 + x^2}\, dx + x\sqrt{1 + y^2}\, dy = 0$
   e. $(1 + x)ydx + (1 + y)xdy = 0$
9. Solve the following differential equations:
   a. $(1 - x^2)(1 - y)\, dx = xy(1 + y)\, dy$
   b. $a(xdy + 2ydx) = xydy$
   c. $\frac{ds}{dx} + x^2 = x^2 e^{3s}$
   d. $\log\left(\frac{dy}{dx}\right) = ax + by$
   e. $y\, dx + (1 + x^2)\tan^{-1} x\, dy = 0$
10. Solve the following differential equations:
    a. $(1 + x^2)\, dy + x\sqrt{1 - y^2}\, dx = 0$
    b. $e^x\sqrt{1 - y^2}\, dx + \frac{y}{x} dy = 0$
    c. $x^2 \frac{dy}{dx} + y = 1$

d. $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

e. (v) $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$

11. (i) if $\frac{dy}{dx} = e^{x+y}$ and it is given that for x = 1, y = 1, the find y when x = -1.

(ii) Find the equation if the curve represented by (y – yx) dx + (x + xy) dy = 0. And Passing through the point (1, 1).

12. Solve $\frac{dy}{dx} = 2x^3 - x$, Given y =1 when x = 0.

13. Solve the following differential equations:

a. $(x+y)^2 \frac{dy}{dx} = a^2$

b. $\frac{dy}{dx} = (4x + y + 1)^2$

c. $\text{Sin}^{-1}\left(\frac{dy}{dx}\right) = x + y$

d. $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$

e. $\frac{dy}{dx} = \frac{x+y+1}{x+y}$

f. (x + 2y) (dx – dy)= dx + dy

ANSWERS

1. $y=\frac{1}{6}x^6 \tan^{-1}(x^3) - \frac{1}{6}x^3 + \frac{1}{6}\tan^{-1}(x^3)+C$

2. y= Sin 2x – x -2 sin x – log | Sec x + tan x|+C

3. log $|1 + y^2|$ +x =C

4. a) $e^x + e^{-y} + \frac{x^3}{3} + C = 0$ b) $e^y - e^x - \frac{x^3}{3} + C = 0$ c) $\frac{y}{1-ay} = C(a + x)$

5 a) $x^2 \log x - \frac{x^2}{2} + \frac{x^2}{2} + C$ b) logy=log x + $\frac{1}{x}$ + C c) tan y =C($e^x - 1$)^3 d) $\log\frac{x}{y} - \left(\frac{1}{x} + \frac{1}{y}\right) = C$

e)log| $1 - y^2$|= log| $1 - x^2$|+ log C

6.(a) $\log(y + 1) = \frac{x^2}{2} + x + C$ (b) log(1 + y^2) (1 + x^2) = log C

7. (a)y – x= log $\frac{x}{y}$ + C

(b) |e^y + 1||sin x |= log C

(c) y tan y + log cos y =x tan x + log cos x + C

(d)sec y = – 2 cos x + C

8. a) $\sec y + 2\cos x = C$ (b) (y-x)= $C(1 + xy)$ (c) (y – x)(3x + 1)=Cx

(d)$\sqrt{1+x^2} + \sqrt{1+y^2}$ = log $\frac{(1+\sqrt{1+x^2})+(1+\sqrt{1+y^2})}{xy}$ + C (e) x + y + log xy = C

9. (a) $x^2 - y^2 - 4y = 2\log x + 4 . \log(1 - y) + C_1$ (b) y – a log y =2a log x + C (c)log (1 –e^{-3s})= x^3 + 3C

(d) $be^{ax} + ae^{-by} = C$ (e) $x = \tan\left(\frac{x}{y}\right) + C$

10. (a) $\sin^{-1} y + \frac{1}{2}\log(1 + x^2) = C$ (b) $(x - 1)e^x = \sqrt{1 - y^2} + C$

(c) $\log(1 - y) = \frac{1}{x} + C$ (d) $\sqrt{3}(x + y + 1) = C(1 - x - y - 2xy)$

(e) $\sqrt{1 + x^2} + \sqrt{1 + y^2} = \log\frac{\left(1 + \sqrt{1+x^2}\right)}{(x)} + C$

11. (a) y=-1 (ii) log xy = x- y

12 $y = \frac{1}{2}x^2(x^2 - 1) + 1$

13.(a) $y = a\tan^{-1}\frac{x+y}{a} + C$ (b) $\frac{1}{2}\tan^{-1}\frac{4x+y+1}{2} = x + C$ (c) $\tan(x + y) - \sec(x + y) = x + C$

d) $\log\left[1 + \tan\frac{x+y}{2}\right] = x + C$ (e) $2(y - x) = \log(2x + 2y + 1) + C$ (f) $3(y - x) + 2\log(3x + 6y - 1) = \frac{9}{2}C_1 = C$.

Assignments- 4 **(NCERT Exercise 9.5 – Homogeneous Differential Equation)**

1. (i) Solve x $\frac{dy}{dx}$ sin $(\frac{y}{x})$ – x – y sin $(\frac{y}{x})$ = 0, y (1) = $\frac{\pi}{2}$.
   (ii) Solve x$e^{\frac{y}{x}}$ + y sin $(\frac{y}{x})$ – x $\frac{dy}{dx}$ sin $(\frac{y}{x})$ =0, y(1) = 0.
2. Solve the differential equation xdx + ydy = m (xdy – ydx).
3. Solve the differential equation (i) x $\frac{dy}{dx}$ = y (log y – log x + 1) (ii) x $\frac{dy}{dx}$ = y – x $\cos^2(\frac{y}{x})$
4. Solve the differential equation $(y^3 - 2yx^2)\,dx + (2xy^2 - x^3)\,dy = 0$.

ANSWERS

1.(i) $-\cos\left(\frac{y}{x}\right) = \log|x|$ (ii) $-e^{-y/x}\left\{\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right\} = \log x^2 + 1$

2. $m\tan^{-1}\frac{y}{x} - \frac{1}{2}\log(x^2+y^2) = C$

3.(i) $\log\frac{y}{x} = Cx$ (ii) $\tan(\frac{y}{x}) + \log x = C$

4. $xy\sqrt{y^2 - x^2} = \sqrt{C_2} = C$

Assignments-5 (NCERT Exercise 9.6- Linear Differential equation, Solution of the type $\frac{dy}{dx}$ +Py=Q and $\frac{dx}{dy}$ +Px = Q)

1. Solve the equation, ydx – xdy + log xdx= 0.
2. Solve the following equations:
   (i) x (x – 1) $\frac{dy}{dx}$ - (x – 2) y = x^3 (2x – 1) (ii) $\frac{dy}{dx}$ + y tan x = x^m cos x .
3. Solve the following differential equation:
   (i) (1 + x^2) $\frac{dy}{dx}$ + 2xy = cos x
   (ii) x log x $\frac{dy}{dx}$ + y = 2 log x.
4. Solve the following equations:
   (i) (x + 2y^3) $\frac{dy}{dx}$ = y. (ii) (1 + y^2) dx = ($\tan^{-1}$ y – x) dy (iii) (1 +y^2) + (x - $e^{tan^{-1}y}$) $\frac{dy}{dx}$ = 0
   (iv) (2x – 10y^3) $\frac{dy}{dx}$ + y = 0

ANSWERS

1.- y +1+log x =C x

2. (i) y(x – 1)= x^2 (x^2- x + C). (ii)y sec x = $\frac{x^{m+1}}{m+1}$ +C.

3. (i) y(1 + x^2) =Sin x + C (ii) y log x = (log x)^2 + C

4. (i) x= y^3 + Cy (ii) $xe^{tan^{-1}y}$ =($\tan^{-1}$y – 1)$e^{tan^{-1}y}$ + C (iii)x. $e^{tan^{-1}y} = \frac{1}{2} e^{2tan^{-1}y}$ + C

(iv) xy^2 = 2y^5 + C