

QUESTION PAPER CODE 65/2/1/F
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. 1×1 1
2. Expanding we get $\frac{1}{2} + \frac{1}{2}$
- $$x^3 = -8 \Rightarrow x = -2$$
3. $P = \frac{1}{2}(A + A')$ $\therefore P = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$ $\frac{1}{2} + \frac{1}{2}$
4. $(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$ $\frac{1}{2}$
- $$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$
- $$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$
5. $a^2 b^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta = 400$ $\frac{1}{2}$
- $$\Rightarrow |\vec{b}| = 4$$
- $\frac{1}{2}$
6. $\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 5\sqrt{3}$ or $x + y + z = 15$ $\left[\frac{1}{2} \text{ mark for dc's of normal} \right]$ 1

SECTION B

7. $\text{LHS} = \cot^{-1} \left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) + \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)}{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) - \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)} \right]$ 1+1
- $$= \cot^{-1} \left(\cot \frac{x}{2} \right)$$
- 1
- $$= \frac{x}{2} = \text{RHS}$$
- 1

OR

- $$\tan^{-1} \left[\frac{\frac{x-2}{x-1} + \frac{x+2}{x+1}}{1 - \frac{x-2}{x-1} \cdot \frac{x+2}{x+1}} \right] = \frac{\pi}{4}$$
- $$\Rightarrow \frac{2x^2 - 4}{3} = \tan \frac{\pi}{4}$$
- $$\Rightarrow x = \pm \sqrt{\frac{7}{2}}$$
- 1

8. Let each poor child pay ₹ x per month and each rich child pay ₹ y per month.

$$\therefore 20x + 5y = 9000$$

 $\frac{1}{2}$

$$5x + 25y = 26000$$

 $\frac{1}{2}$

In matrix form,

$$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$

1

$$AX = B \Rightarrow X = A^{-1} B$$

$$A^{-1} = \frac{1}{475} \begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475} \begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix} \begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$$\Rightarrow x = 200, y = 1000$$

1

Value: Compassion or any relevant value

1

9. $f'_{1-} = 2x + 3 = 5$

$$f'_{1+} = b$$

$$f'_{1-} = f'_{1+} \Rightarrow \boxed{b = 5}$$

1+1

$$\lim_{x \rightarrow 1^-} f(x) = f(1) = \lim_{x \rightarrow 1^+} f(x)$$

$$\Rightarrow 4 + a = b + 2$$

1

$$\Rightarrow \boxed{a = 3}$$

1

10. Let $u = \tan^{-1} \frac{\sqrt{1+x^2} - 1}{x}$

Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$

 $\frac{1}{2}$

$$\therefore u = \tan^{-1} \left[\frac{\sec \theta - 1}{\tan \theta} \right]$$

$$= \tan^{-1} \left[\frac{1 - \cos \theta}{\sin \theta} \right]$$

$$= \tan^{-1} \left(\tan \frac{\theta}{2} \right)$$

$$= \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$

1

$$\Rightarrow \frac{du}{dx} = \frac{1}{2(1+x^2)}$$

1

$$v = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$= 2 \tan^{-1} x$$

$$\Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2} \quad 1$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{4} \quad \frac{1}{2}$$

OR

$$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t \quad \frac{1}{2}$$

$$y = \sin pt \Rightarrow \frac{dy}{dt} = p \cos pt \quad \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{p \cos pt}{\cos t} \quad 1$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{\cos t (-p^2 \sin pt) - p \cos pt (-\sin t)}{\cos^2 t} \cdot \frac{dt}{dx} \\ &= \frac{-p^2 \sin pt \cos t + p \cos pt \sin t}{\cos^3 t} \quad 1 \end{aligned}$$

$$\text{Now } (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0 \left[\text{Substituting values of } y, \frac{dy}{dx} \text{ \& } \frac{d^2y}{dx^2} \right] \quad 1$$

11. Eqn of given curves

$$y^2 = 4ax \text{ and } x^2 = 4by$$

Their point of intersections are $(0, 0)$ and $(4a^{1/3} b^{2/3}, 4a^{2/3} b^{1/3})$ 1

$$y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y}, \text{ slope} = \frac{a^{1/3}}{2b^{1/3}} \quad \dots(i) \quad 1$$

$$x^2 = 4by \Rightarrow \frac{dy}{dx} = \frac{x}{2b}, \text{ slope} = \frac{2a^{1/3}}{b^{1/3}} \quad \dots(ii) \quad 1$$

At $(0, 0)$, angle between two curves is 90°

or

Acute angle θ between (i) and (ii) is

$$\theta = \tan^{-1} \left\{ \frac{3 \left(\frac{a^{1/3} b^{1/3}}{a^{2/3} + b^{2/3}} \right)}{2} \right\}$$

12. $I = \int_0^\pi \frac{(\pi-x)}{1+\sin \alpha \sin (\pi-x)} dx$ 1

$$2I = \pi \int_0^\pi \frac{dx}{1+\sin \alpha \sin x}$$

$$= 2\pi \int_0^{\pi/2} \frac{dx}{1+\sin \alpha \sin x}$$

$$\Rightarrow I = \pi \int_0^{\pi/2} \frac{dx}{1+\sin \alpha \frac{2 \tan \frac{x}{2}}{1+\tan^2 \frac{x}{2}}} \quad 1$$

$$I = \pi \int_0^1 \frac{2dt}{1+t^2+2t \sin \alpha} \quad \text{Put } \tan \frac{x}{2} = t \quad \frac{1}{2}$$

$$\Rightarrow I = 2\pi \int_0^1 \frac{dt}{(t + \sin \alpha)^2 + \cos^2 \alpha} \quad 1$$

$$= \frac{2\pi}{\cos \alpha} \left[\tan^{-1} \left(\frac{t + \sin \alpha}{\cos \alpha} \right) \right]_0^1$$

$$\Rightarrow I = \frac{\pi}{\cos \alpha} \left(\frac{\pi}{2} - \alpha \right) \quad \frac{1}{2}$$

13. $I = \int (2x+5)\sqrt{10-4x-3x^2} dx$

$$= -\frac{1}{3} \int (-4-6x)\sqrt{10-4x-3x^2} dx + \frac{11}{3} \int \sqrt{10-4x-3x^2} dx \quad 1$$

$$= -\frac{2}{9} (10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3} \int \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x-\frac{2}{3}\right)^2} dx \quad 1+1$$

$$= -\frac{2}{9} (10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3} \left[\frac{\left(x-\frac{2}{3}\right) - \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x-\frac{2}{3}\right)^2}}{2} + \frac{17}{9} \sin^{-1} \frac{3x-2}{\sqrt{34}} \right] + C \quad 1$$

OR

$$x^2 = y \text{ (say)} \quad \frac{1}{2}$$

$$\frac{(y+1)(y+4)}{(y+3)(y-5)} = 1 + \frac{A}{y+3} + \frac{B}{y-5} \quad \frac{1}{2}$$

using partial fraction we get $A = \frac{1}{4}$, $B = \frac{27}{4}$ 1

$$\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)} dx = \int 1 dx + \frac{1}{4} \int \frac{dx}{x^2+3} + \frac{27}{4} \int \frac{dx}{x^2-5} \quad 1$$

$$= x + \frac{1}{4\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}} \log \left| \frac{x-\sqrt{5}}{x+\sqrt{5}} \right| + C \quad 1$$

14. $I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

$$\text{put } \sin^{-1} x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt \quad \frac{1}{2} + \frac{1}{2}$$

$$= \int t \cdot \sin t dt \quad 1$$

$$= -t \cos t + \sin t + c \quad \frac{1}{2}$$

$$= -\sqrt{1-x^2} \sin^{-1} x + x + c \quad \frac{1}{2}$$

$$15. \quad y^2 dx + (x^2 - xy + y^2) dy = 0$$

$$\Rightarrow \frac{dx}{dy} = -\frac{(x^2 - xy + y^2)}{y^2} \quad \frac{1}{2}$$

$$\text{put } x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy} \quad \frac{1}{2}$$

$$v + y \frac{dv}{dy} = \frac{(v^2 y^2 - y^2 v + y^2)}{y^2}$$

$$\Rightarrow \frac{dv}{v^2 + 1} = -\frac{dy}{y} \quad 1 \frac{1}{2}$$

Integrating both sides

$$\tan^{-1} v = -\log y + c \quad \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \tan^{-1} \frac{x}{y} = -\log y + c \quad \frac{1}{2}$$

$$16. \quad \frac{dx}{dy} - \frac{x}{1+y^2} = \frac{\cot^{-1} y}{1+y^2} \quad \frac{1}{2}$$

$$\text{I.F} = e^{-\int \frac{1}{1+y^2}} = e^{\cot^{-1} y} \quad 1$$

$$\Rightarrow \frac{d}{dy} (x \cdot e^{\cot^{-1} y}) = \frac{\cot^{-1} y \cdot e^{\cot^{-1} y}}{1+y^2}$$

Integrating, we get

$$x \cdot e^{\cot^{-1} y} = \int \frac{\cot^{-1} y \cdot e^{\cot^{-1} y}}{1+y^2} dy \quad 1 \frac{1}{2}$$

put $\cot^{-1} y = t$

$$= -\int t e^t dt$$

$$= (1-t) e^t + c$$

$$\Rightarrow x = (1 - \cot^{-1} y) + c e^{-\cot^{-1} y} \quad 1$$

$$17. \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{d} \quad \dots(\text{i})$$

$$\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \quad \dots(\text{ii})$$

$$(1) - (2) \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{d} \times (\vec{b} - \vec{c}) \quad 2$$

$$\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0} \quad 1$$

$$\Rightarrow (\vec{a} - \vec{d}) \parallel (\vec{b} - \vec{c}) \quad 1$$

18. Equation of line \overline{AB}

$$\vec{r} = (-\hat{j} - \hat{k}) + \lambda (4\hat{i} + 6\hat{j} + 2\hat{k}) \quad 1$$

Equation of line \overline{CD}

$$\vec{r} = (3\hat{i} + 9\hat{j} + 4\hat{k}) + \mu (-7\hat{i} - 5\hat{j}) \quad \frac{1}{2}$$

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 10\hat{j} + 5\hat{k} \quad \frac{1}{2}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 10\hat{i} - 14\hat{j} + 22\hat{k} \quad 1$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 30 - 140 + 110 = 0 \quad 1$$

\Rightarrow Lines intersect

19. Let selection of defective pen be considered success

$$p = \frac{2}{20} = \frac{1}{10}, q = \frac{9}{10} \quad 1$$

$$\text{Reqd probability} = P(x=0) + P(x=1) + P(x=2) \quad 1\frac{1}{2}$$

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 + {}^5C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2} \left(\frac{9}{10}\right)^4 + \frac{1}{10} \times \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^3 \times \frac{34}{25} \quad 1\frac{1}{2}$$

OR

$$\sum_{i=0}^4 P(x_i) = 1 \quad \frac{1}{2}$$

$$\Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8} \quad \frac{1}{2}$$

$$(i) P(x=1) = \frac{1}{8} \quad 1$$

$$(ii) P(\text{at most 2 colleges}) = P(0) + P(1) + P(2)$$

$$= \frac{5}{8} \quad 1$$

$$(iii) P(\text{at least 2 colleges}) = 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \frac{1}{8} = \frac{7}{8} \quad 1$$

20. $f(x) = |x| + x, \quad g(x) = |x| - x \quad \forall x \in \mathbb{R}$

$(f \circ g)(x) = f(g(x))$ 1

$= ||x| - 1| + |x| - x$ 1

$(g \circ f)(x) = g(f(x))$ $\frac{1}{2}$

$= ||x| + x| - |x| - x$ $\frac{1}{2}$

$(f \circ g)(-3) = 6$ 1

$(f \circ g)(5) = 0$ 1

$(g \circ f)(-2) = 2$ 1

21. $abc \begin{vmatrix} \frac{1}{a} + 1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} + 1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$C_1 \rightarrow C_1 + C_2 + C_3$

$\Rightarrow abc \begin{vmatrix} 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} + 1 & \frac{1}{c} \\ 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$\Rightarrow abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b} + 1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0$

$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$ $1\frac{1}{2}$

$\Rightarrow abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = 0$

$\therefore a, b, c, \neq 0$

$\therefore 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$ $1\frac{1}{2}$

OR

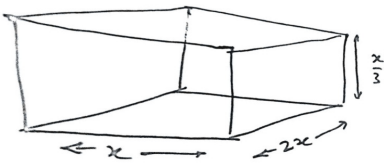
$$|A| = 1$$

$$\text{adj } A = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A(\text{adj } A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$|A| I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

22.



$$S = 6x^2 + 4\pi r^2$$

$$\Rightarrow r = \sqrt{\frac{S - 6x^2}{4\pi}} \quad \dots(i)$$

$$V = \frac{2x^3}{3} + \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}x^3 + \frac{4}{3}\pi \left(\frac{S - 6x^2}{4\pi} \right)^{3/2}$$

$$= \frac{2x^3}{3} + \frac{(S - 6x^2)^{3/2}}{6\sqrt{\pi}}$$

$$\frac{dV}{dx} = 2x^2 - \frac{3x}{\sqrt{\pi}} \sqrt{S - 6x^2}$$

$$\frac{dV}{dx} = 0 \Rightarrow 2x^2 = \frac{3x \sqrt{S - 6x^2}}{\sqrt{\pi}}$$

$$\Rightarrow r = \frac{x}{3} \text{ [using (i)]}$$

$$\left. \begin{aligned} \frac{d^2V}{dx^2} &= 4x \left[\frac{3x}{\sqrt{\pi}} \cdot \frac{(-12x)}{2\sqrt{S - 6x^2}} + \frac{3}{\sqrt{\pi}} \sqrt{S - 6x^2} \right] \\ \frac{d^2V}{dx^2} \Big|_{x=\frac{r}{3}} &> 0 \end{aligned} \right\}$$

$$\Rightarrow V \text{ is minimum at } x = \frac{r}{3} \text{ i.e. } r = 3x$$

$$\text{Minimum value of sum of volume} = \left(\frac{2x^3}{3} + 36\pi x^3 \right) \text{ cubic units}$$

OR

Equation of given curve

$$y = \cos(x + y) \quad \dots(i)$$

$$\Rightarrow \frac{dy}{dx} = -\sin(x + y) \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x + y)}{1 + \sin(x + y)} \quad 2$$

given line $x + 2y = 0$, its slope $= -\frac{1}{2}$ 1/2

condition of || lines

$$\frac{-\sin(x + y)}{1 + \sin(x + y)} = -\frac{1}{2} \quad 1$$

$$\Rightarrow \sin(x + y) = 1$$

$$\Rightarrow \cos(x + y) = 0 \quad y = 0 \quad \text{using (i)}$$

$$\Rightarrow \cos x = 0 \Rightarrow x = (2n + 1) \frac{\pi}{2}, n \in \mathbb{I}$$

$$\therefore x = \frac{-3\pi}{2}, \frac{\pi}{2} \in [-2\pi, 2\pi] \quad 1$$

Thus tangents are || to the line $x + 2y = 0$

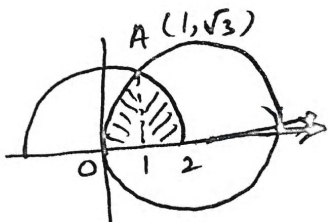
only at pts $\left(-\frac{3\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 0\right)$ 1/2

 \therefore Required equation of tangents are

$$y - 0 = -\frac{1}{2} \left(x + \frac{3\pi}{2}\right) \Rightarrow 2x + 4y + 3\pi = 0 \quad 1/2$$

$$y - 0 = -\frac{1}{2} \left(x - \frac{\pi}{2}\right) \Rightarrow 2x + 4y - \pi = 0 \quad 1/2$$

23. Their point of intersection $(1, \sqrt{3})$ 1

Correct Figure 1

$$\text{Required Area} = \int_0^1 \sqrt{(2)^2 - (x-2)^2} dx + \int_1^2 \sqrt{2^2 - x^2} dx \quad 2$$

$$= \left[\frac{(x-2)\sqrt{4x-x^2}}{2} + 2 \sin^{-1} \frac{x-2}{2} \right]_0^1 + \left[\frac{x\sqrt{4-x^2}}{2} + 2 \sin^{-1} \frac{x}{2} \right]_1^2 \quad 1$$

$$= \left(\frac{5\pi}{3} - \sqrt{3} \right) \text{Sq. units} \quad 1$$

24. Equation of family of planes passing through two given planes

$$(x + 2y + 3z - 4) + k(2x + y - z + 5) = 0 \quad 1$$

$$\Rightarrow (1 + 2k)x + (2 + k)y + (3 - k)z = 4 - 5k \quad \dots(i)$$

$$\Rightarrow \frac{x}{\frac{4-5k}{1+2k}} + \frac{y}{\frac{4-5k}{2+k}} + \frac{z}{\frac{4-5k}{3-k}} = 1$$

As per condition

$$\frac{4-5k}{1+2k} = \frac{2(4-5k)}{(3-k)} \quad 1$$

$$\Rightarrow k = \frac{4}{5} \text{ or } \frac{1}{5} \quad 1$$

For $k = \frac{1}{5}$, Eqn. of plane is $7x + 11y + 14z = 15$ 1

For $k = \frac{4}{5}$, Eqn. of plane is $13x + 14y + 11z = 0$ $\frac{1}{2}$

Equation of plane passing through $(2, 3, -1)$

and parallel to the plane is:

$$7(x - 2) + 11(y - 3) + 14(z + 1) = 0 \quad 1$$

$$\Rightarrow 7x + 11y + 14z = 33$$

Vector form: $\vec{r} \cdot (7\hat{i} + 11\hat{j} + 14\hat{k}) = 33$ $\frac{1}{2}$

25. Let H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 black ball, transferred

H_3 be the event 2 black and 1 black ball transferred

E be the event that ball drawn from B is red. 1

$$P(H_1) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28} \quad P(E/H_1) = \frac{6}{10}$$

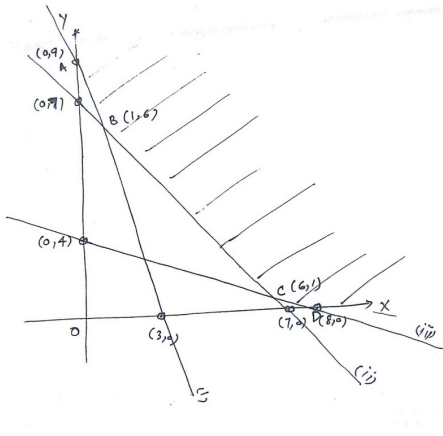
$$P(H_2) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28} \quad P(E/H_2) = \frac{5}{10}$$

$$P(H_3) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28} \quad P(E/H_3) = \frac{4}{10} \quad 1\frac{1}{2} + 1\frac{1}{2}$$

$$P(H_1/E) = \frac{\frac{3}{28} \times \frac{6}{10}}{\frac{3}{28} \times \frac{6}{10} + \frac{15}{28} \times \frac{5}{10} + \frac{10}{28} \times \frac{4}{10}} \quad 1\frac{1}{2}$$

$$= \frac{18}{133} \quad 1\frac{1}{2}$$

26.

Let x tablets of type X and y tablets of type Y are takenMinimise $C = 2x + y$

subjected to

$$\left. \begin{aligned} 6x + 2y &\geq 18 \\ 3x + 3y &\geq 21 \\ 2x + 4y &\geq 16 \\ x, y &\geq 0 \end{aligned} \right\}$$

Correct Graph

$$Cl_{A(0,9)} = 9$$

$$Cl_{B(1,6)} = 8 \leftarrow \text{Minimum value}$$

$$Cl_{C(6,1)} = 13$$

$$Cl_{D(8,0)} = 16$$

 $2x + y < 8$ does not pass through unbounded regionThus, minimum value of $C = 8$ at $x = 1, y = 6$.

1

2

 $1\frac{1}{2}$ $\frac{1}{2}$

1

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SECTION A

1. $P = \frac{1}{2}(A + A') \quad \therefore P = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$ $\frac{1}{2} + \frac{1}{2}$
2. $(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$ $\frac{1}{2}$
- $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$
- $\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$ $\frac{1}{2}$
3. $a^2 b^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta = 400$ $\frac{1}{2}$
- $\Rightarrow |\vec{b}| = 4$ $\frac{1}{2}$
4. $\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 5\sqrt{3}$ or $x + y + z = 15$ $\left[\frac{1}{2} \text{ mark for dc's of normal} \right]$ 1
5. 1×1 1
6. Expanding we get
- $x^3 = -8 \Rightarrow x = -2$ $\frac{1}{2} + \frac{1}{2}$

SECTION B

7. $f'_{1-} = 2x + 3 = 5$
- $f'_{1+} = b$
- $f'_{1-} = f'_{1+} \Rightarrow \boxed{b=5}$ 1+1
- $\lim_{x \rightarrow 1^-} f(x) = f(1) = \lim_{x \rightarrow 1^+} f(x)$
- $\Rightarrow 4 + a = b + 2$ 1
- $\Rightarrow \boxed{a=3}$ 1
8. Let $u = \tan^{-1} \frac{\sqrt{1+x^2} - 1}{x}$
- Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$ $\frac{1}{2}$
- $\therefore u = \tan^{-1} \left[\frac{\sec \theta - 1}{\tan \theta} \right]$
- $= \tan^{-1} \left[\frac{1 - \cos \theta}{\sin \theta} \right]$

$$= \tan^{-1} \left(\tan \frac{\theta}{2} \right)$$

$$= \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2(1+x^2)}$$

$$v = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$= 2 \tan^{-1} x$$

$$\Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{4}$$

OR

$$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t$$

$$y = \sin pt \Rightarrow \frac{dy}{dt} = p \cos pt$$

$$\frac{dy}{dx} = \frac{p \cos pt}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{\cos t (-p^2 \sin pt) - p \cos pt (-\sin t)}{\cos^2 t} \cdot \frac{dt}{dx}$$

$$= \frac{-p^2 \sin pt \cos t + p \cos pt \sin t}{\cos^3 t}$$

$$\text{Now } (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0 \left[\text{Substituting values of } y, \frac{dy}{dx} \text{ \& } \frac{d^2y}{dx^2} \right]$$

9. Eqn of given curves

$$y^2 = 4ax \text{ and } x^2 = 4by$$

Their point of intersections are (0, 0) and $(4a^{1/3} b^{2/3}, 4a^{2/3} b^{1/3})$

$$y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y}, \text{ slope} = \frac{a^{1/3}}{2b^{1/3}} \quad \dots(i)$$

$$x^2 = 4by \Rightarrow \frac{dy}{dx} = \frac{x}{2b}, \text{ slope} = \frac{2a^{1/3}}{b^{1/3}} \quad \dots(ii)$$

At (0, 0), angle between two curves is 90°

or

Acute angle θ between (i) and (ii) is

$$\theta = \tan^{-1} \left\{ \frac{3 \left(\frac{a^{1/3} b^{1/3}}{a^{2/3} + b^{2/3}} \right)}{2} \right\}$$

$$10. \quad I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin \alpha \sin (\pi - x)} dx \quad 1$$

$$2I = \pi \int_0^{\pi} \frac{dx}{1 + \sin \alpha \sin x}$$

$$= 2\pi \int_0^{\pi/2} \frac{dx}{1 + \sin \alpha \sin x}$$

$$\Rightarrow I = \pi \int_0^{\pi/2} \frac{dx}{1 + \sin \alpha \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} \quad 1$$

$$I = \pi \int_0^1 \frac{2dt}{1 + t^2 + 2t \sin \alpha} \quad \text{Put } \tan \frac{x}{2} = t \quad \frac{1}{2}$$

$$\Rightarrow I = 2\pi \int_0^1 \frac{dt}{(t + \sin \alpha)^2 + \cos^2 \alpha} \quad 1$$

$$= \frac{2\pi}{\cos \alpha} \left[\tan^{-1} \left(\frac{t + \sin \alpha}{\cos \alpha} \right) \right]_0^1$$

$$\Rightarrow I = \frac{\pi}{\cos \alpha} \left(\frac{\pi}{2} - \alpha \right) \quad \frac{1}{2}$$

$$11. \quad I = \int (2x + 5) \sqrt{10 - 4x - 3x^2} dx$$

$$= -\frac{1}{3} \int (-4 - 6x) \sqrt{10 - 4x - 3x^2} dx + \frac{11}{3} \int \sqrt{10 - 4x - 3x^2} dx \quad 1$$

$$= -\frac{2}{9} (10 - 4x - 3x^2)^{3/2} + \frac{11\sqrt{3}}{3} \int \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x - \frac{2}{3}\right)^2} dx \quad 1 + 1$$

$$= -\frac{2}{9} (10 - 4x - 3x^2)^{3/2} + \frac{11\sqrt{3}}{3} \left[\frac{\left(x - \frac{2}{3}\right) - \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x - \frac{2}{3}\right)^2}}{2} + \frac{17}{9} \sin^{-1} \frac{3x - 2}{\sqrt{34}} \right] + C \quad 1$$

OR

$$x^2 = y \text{ (say)} \quad \frac{1}{2}$$

$$\frac{(y+1)(y+4)}{(y+3)(y-5)} = 1 + \frac{A}{y+3} + \frac{B}{y-5} \quad \frac{1}{2}$$

$$\text{using partial fraction we get } A = \frac{1}{4}, B = \frac{27}{4} \quad 1$$

$$\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)} dx = \int 1 dx + \frac{1}{4} \int \frac{dx}{x^2+3} + \frac{27}{4} \int \frac{dx}{x^2-5} \quad 1$$

$$= x + \frac{1}{4\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}} \log \left| \frac{x - \sqrt{5}}{x + \sqrt{5}} \right| + C \quad 1$$

$$12. \quad I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$\text{put } \sin^{-1} x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt \quad \frac{1}{2} + \frac{1}{2}$$

$$= \int t \cdot \sin t dt \quad 1$$

$$= -t \cos t + \sin t + c \quad 1 \frac{1}{2}$$

$$= -\sqrt{1-x^2} \sin^{-1} x + x + c \quad \frac{1}{2}$$

$$13. \quad y^2 dx + (x^2 - xy + y^2) dy = 0$$

$$\Rightarrow \frac{dx}{dy} = -\frac{(x^2 - xy + y^2)}{y^2} \quad \frac{1}{2}$$

$$\text{put } x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy} \quad \frac{1}{2}$$

$$v + y \frac{dv}{dy} = \frac{(v^2 y^2 - y^2 v + y^2)}{y^2}$$

$$\Rightarrow \frac{dv}{v^2 + 1} = -\frac{dy}{y} \quad 1 \frac{1}{2}$$

Integrating both sides

$$\tan^{-1} v = -\log y + c \quad \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \tan^{-1} \frac{x}{y} = -\log y + c \quad \frac{1}{2}$$

$$14. \quad \frac{dx}{dy} - \frac{x}{1+y^2} = \frac{\cot^{-1} y}{1+y^2} \quad \frac{1}{2}$$

$$\text{I.F} = e^{-\int \frac{1}{1+y^2}} = e^{\cot^{-1} y} \quad 1$$

$$\Rightarrow \frac{d}{dy} (x \cdot e^{\cot^{-1} y}) = \frac{\cot^{-1} y e^{\cot^{-1} y}}{1+y^2}$$

Integrating, we get

$$x \cdot e^{\cot^{-1} y} = \int \frac{\cot^{-1} y e^{\cot^{-1} y}}{1+y^2} dy \quad 1 \frac{1}{2}$$

$$\text{put } \cot^{-1} y = t$$

$$= -\int t e^t dt$$

$$= (1-t) e^t + c$$

$$\Rightarrow x = (1 - \cot^{-1} y) + ce^{-\cot^{-1} y} \quad 1$$

$$15. \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{d} \quad \dots(i)$$

$$\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \quad \dots(ii)$$

$$(1) - (2) \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{d} \times (\vec{b} - \vec{c}) \quad 2$$

$$\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0} \quad 1$$

$$\Rightarrow (\vec{a} - \vec{d}) \parallel (\vec{b} - \vec{c}) \quad 1$$

16. Equation of line \overline{AB}

$$\vec{r} = (-\hat{j} - \hat{k}) + \lambda (4\hat{i} + 6\hat{j} + 2\hat{k}) \quad 1$$

Equation of line \overline{CD}

$$\vec{r} = (3\hat{i} + 9\hat{j} + 4\hat{k}) + \mu (-7\hat{i} - 5\hat{j}) \quad \frac{1}{2}$$

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 10\hat{j} + 5\hat{k} \quad \frac{1}{2}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 10\hat{i} - 14\hat{j} + 22\hat{k} \quad 1$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 30 - 140 + 110 = 0 \quad 1$$

\Rightarrow Lines intersect

17. Let selection of defective pen be considered success

$$p = \frac{2}{20} = \frac{1}{10}, q = \frac{9}{10} \quad 1$$

$$\text{Reqd probability} = P(x=0) + P(x=1) + P(x=2) \quad 1\frac{1}{2}$$

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 + {}^5C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2} \left(\frac{9}{10}\right)^4 + \frac{1}{10} \times \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^3 \times \frac{34}{25} \quad 1\frac{1}{2}$$

OR

$$\sum_{i=0}^4 P(x_i) = 1 \quad \frac{1}{2}$$

$$\Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8} \quad \frac{1}{2}$$

$$(i) P(x=1) = \frac{1}{8} \quad 1$$

$$(ii) P(\text{at most 2 colleges}) = P(0) + P(1) + P(2)$$

$$= \frac{5}{8} \quad 1$$

$$(iii) P(\text{at least 2 colleges}) = 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \frac{1}{8} = \frac{7}{8} \quad 1$$

$$18. \text{ LHS} = \cot^{-1} \left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) + \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)}{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) - \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)} \right] \quad 1+1$$

$$= \cot^{-1} \left(\cot \frac{x}{2} \right) \quad 1$$

$$= \frac{x}{2} = \text{RHS} \quad 1$$

OR

$$\tan^{-1} \left[\frac{\frac{x-2}{x-1} + \frac{x+2}{x+1}}{1 - \frac{x-2}{x-1} \cdot \frac{x+2}{x+1}} \right] = \frac{\pi}{4} \quad 1 \frac{1}{2}$$

$$\Rightarrow \frac{2x^2 - 4}{3} = \tan \frac{\pi}{4} \quad 1 \frac{1}{2}$$

$$\Rightarrow x = \pm \sqrt{\frac{7}{2}} \quad 1$$

19. Let each poor child pay ₹ x per month and each rich child pay ₹ y per month.

$$\therefore 20x + 5y = 9000 \quad \frac{1}{2}$$

$$5x + 25y = 26000 \quad \frac{1}{2}$$

In matrix form,

$$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix} \quad 1$$

$$AX = B \Rightarrow X = A^{-1} B$$

$$A^{-1} = \frac{1}{475} \begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

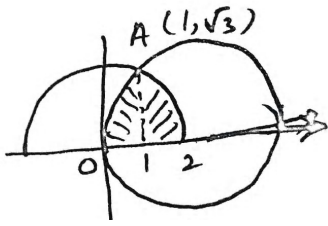
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475} \begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix} \begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$$\Rightarrow x = 200, y = 1000 \quad 1$$

Value: Compassion or any relevant value 1

20. Their point of intersection $(1, \sqrt{3})$ 1

Correct Figure 1



$$\text{Required Area} = \int_0^1 \sqrt{(2)^2 - (x-2)^2} dx + \int_1^2 \sqrt{2^2 - x^2} dx \quad 2$$

$$= \left[\frac{(x-2)\sqrt{4x-x^2}}{2} + 2 \sin^{-1} \frac{x-2}{2} \right]_0^1 + \left[\frac{x\sqrt{4-x^2}}{2} + 2 \sin^{-1} \frac{x}{2} \right]_1^2 \quad 1$$

$$= \left(\frac{5\pi}{3} - \sqrt{3} \right) \text{Sq. units} \quad 1$$

21. Equation of family of planes passing through two given planes

$$(x + 2y + 3z - 4) + k(2x + y - z + 5) = 0 \quad 1$$

$$\Rightarrow (1 + 2k)x + (2 + k)y + (3 - k)z = 4 - 5k \quad \dots(i)$$

$$\Rightarrow \frac{x}{\frac{4-5k}{1+2k}} + \frac{y}{\frac{4-5k}{2+k}} + \frac{z}{\frac{4-5k}{3-k}} = 1$$

As per condition

$$\frac{4-5k}{1+2k} = \frac{2(4-5k)}{3-k} \quad 1$$

$$\Rightarrow k = \frac{4}{5} \text{ or } \frac{1}{5} \quad 1$$

For $k = \frac{1}{5}$, Eqn. of plane is $7x + 11y + 14z = 15$ 1

For $k = \frac{4}{5}$, Eqn. of plane is $13x + 14y + 11z = 0$ $\frac{1}{2}$

Equation of plane passing through $(2, 3, -1)$

and parallel to the plane is:

$$7(x-2) + 11(y-3) + 14(z+1) = 0 \quad 1$$

$$\Rightarrow 7x + 11y + 14z = 33$$

Vector form: $\vec{r} \cdot (7\hat{i} + 11\hat{j} + 14\hat{k}) = 33$ $\frac{1}{2}$

22. Let H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 black ball, transferred

H_3 be the event 2 black and 1 black ball transferred

E be the event that ball drawn from B is red. 1

$$P(H_1) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28} \quad P(E/H_1) = \frac{6}{10}$$

$$P(H_2) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28} \quad P(E/H_2) = \frac{5}{10}$$

$$P(H_3) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28}$$

$$P(E/H_3) = \frac{4}{10}$$

$$1\frac{1}{2} + 1\frac{1}{2}$$

$$P(H_1/E) = \frac{\frac{3}{28} \times \frac{6}{10}}{\frac{3}{28} \times \frac{6}{10} + \frac{15}{28} \times \frac{5}{10} + \frac{10}{28} \times \frac{4}{10}}$$

$$= \frac{18}{133}$$

$$1\frac{1}{2}$$

$$\frac{1}{2}$$

23.

Let x tablets of type X and y tablets of type Y are taken

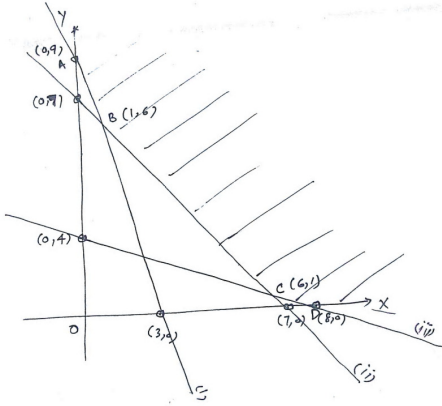
$$\text{Minimise } C = 2x + y$$

1

subjected to

$$\left. \begin{aligned} 6x + 2y &\geq 18 \\ 3x + 3y &\geq 21 \\ 2x + 4y &\geq 16 \\ x, y &\geq 0 \end{aligned} \right\}$$

2



Correct Graph

$$1\frac{1}{2}$$

$$Cl_{A(0,9)} = 9$$

$$Cl_{B(1,6)} = 8 \leftarrow \text{Minimum value}$$

$$Cl_{C(6,1)} = 13$$

$$Cl_{D(8,0)} = 16$$

$2x + y < 8$ does not pass through unbounded region

$$\frac{1}{2}$$

Thus, minimum value of $C = 8$ at $x = 1, y = 6$.

1

24. $f(x) = |x| + x, \quad g(x) = |x| - x \quad \forall x \in \mathbb{R}$

$$(f \circ g)(x) = f(g(x))$$

1

$$= ||x| - 1| + |x| - x$$

1

$$(g \circ f)(x) = g(f(x))$$

$$\frac{1}{2}$$

$$= ||x| + x| - |x| - x$$

$$\frac{1}{2}$$

$$(f \circ g)(-3) = 6$$

1

$$(f \circ g)(5) = 0$$

1

$$(g \circ f)(-2) = 2$$

1

25.
$$abc \begin{vmatrix} \frac{1}{a} + 1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} + 1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0$$

$$1\frac{1}{2}$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$\Rightarrow abc \begin{vmatrix} 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} + 1 & \frac{1}{c} \\ 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0 \quad 1\frac{1}{2}$$

$$\Rightarrow abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b} + 1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1 \quad 1\frac{1}{2}$$

$$\Rightarrow abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = 0$$

$$\therefore a, b, c, \neq 0$$

$$\therefore 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \quad 1\frac{1}{2}$$

OR

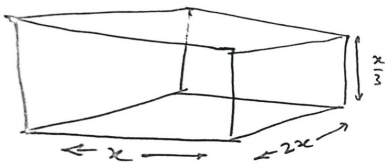
$$|A| = 1 \quad 1$$

$$\text{adj } A = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 3$$

$$A(\text{adj } A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad 1$$

$$|A| I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad 1$$

26.



$$S = 6x^2 + 4\pi r^2$$

$$\Rightarrow r = \sqrt{\frac{S - 6x^2}{4\pi}} \quad \dots(i) \quad 1\frac{1}{2}$$

$$V = \frac{2x^3}{3} + \frac{4}{3}\pi r^3 \quad 1$$

$$= \frac{2}{3}x^3 + \frac{4}{3}\pi \left(\frac{S - 6x^2}{4\pi} \right)^{3/2}$$

$$= \frac{2x^3}{3} + \frac{(S - 6x^2)^{3/2}}{6\sqrt{\pi}} \quad 1$$

$$\frac{dV}{dx} = 2x^2 - \frac{3x}{\sqrt{\pi}} \sqrt{S - 6x^2} \quad 1$$

$$\frac{dV}{dx} = 0 \Rightarrow 2x^2 = \frac{3x \sqrt{S - 6x^2}}{\sqrt{\pi}}$$

$$\Rightarrow r = \frac{x}{3} \text{ [using (i)]} \quad 1$$

$$\left. \begin{aligned} \frac{d^2V}{dx^2} &= 4x \left[\frac{3x}{\sqrt{\pi}} \cdot \frac{(-12x)}{2\sqrt{S-6x^2}} + \frac{3}{\sqrt{\pi}} \sqrt{S-6x^2} \right] \\ \frac{d^2V}{dx^2} \Big|_{x=\frac{r}{3}} &> 0 \end{aligned} \right\} \quad 1$$

$$\Rightarrow V \text{ is minimum at } x = \frac{r}{3} \text{ i.e. } r = 3x$$

$$\text{Minimum value of sum of volume} = \left(\frac{2x^3}{3} + 36\pi x^3 \right) \text{ cubic units} \quad \frac{1}{2}$$

OR

Equation of given curve

$$y = \cos(x + y) \quad \dots(i)$$

$$\Rightarrow \frac{dy}{dx} = -\sin(x + y) \left(1 + \frac{dy}{dx} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x + y)}{1 + \sin(x + y)} \quad 2$$

$$\text{given line } x + 2y = 0, \text{ its slope} = -\frac{1}{2} \quad \frac{1}{2}$$

condition of || lines

$$\frac{-\sin(x + y)}{1 + \sin(x + y)} = -\frac{1}{2} \quad 1$$

$$\Rightarrow \sin(x + y) = 1$$

$$\Rightarrow \cos(x + y) = 0 \quad y = 0 \quad \text{using (i)}$$

$$\Rightarrow \cos x = 0 \Rightarrow x = (2n + 1) \frac{\pi}{2}, n \in \mathbb{I}$$

$$\therefore x = \frac{-3\pi}{2}, \frac{\pi}{2} \in [-2\pi, 2\pi] \quad 1$$

Thus tangents are || to the line $x + 2y = 0$

$$\text{only at pts } \left(-\frac{3\pi}{2}, 0 \right) \text{ and } \left(\frac{\pi}{2}, 0 \right) \quad \frac{1}{2}$$

\therefore Required equation of tangents are

$$y - 0 = -\frac{1}{2} \left(x + \frac{3\pi}{2} \right) \Rightarrow 2x + 4y + 3\pi = 0 \quad \frac{1}{2}$$

$$y - 0 = -\frac{1}{2} \left(x - \frac{\pi}{2} \right) \Rightarrow 2x + 4y - \pi = 0 \quad \frac{1}{2}$$

QUESTION PAPER CODE 65/2/3/F
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. $(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$ $\frac{1}{2}$
 $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$
 $\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$ $\frac{1}{2}$
2. $a^2 b^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta = 400$ $\frac{1}{2}$
 $\Rightarrow |\vec{b}| = 4$ $\frac{1}{2}$
3. $\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 5\sqrt{3}$ or $x + y + z = 15$ $\left[\frac{1}{2} \text{ mark for dc's of normal} \right]$ 1
4. 1×1 1
5. Expanding we get
 $x^3 = -8 \Rightarrow x = -2$ $\frac{1}{2} + \frac{1}{2}$
6. $P = \frac{1}{2}(A + A')$ $\therefore P = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$ $\frac{1}{2} + \frac{1}{2}$

SECTION B

7. Let $u = \tan^{-1} \frac{\sqrt{1+x^2} - 1}{x}$
- Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$ $\frac{1}{2}$
- $\therefore u = \tan^{-1} \left[\frac{\sec \theta - 1}{\tan \theta} \right]$
 $= \tan^{-1} \left[\frac{1 - \cos \theta}{\sin \theta} \right]$
 $= \tan^{-1} \left(\tan \frac{\theta}{2} \right)$
 $= \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$ 1
- $\Rightarrow \frac{du}{dx} = \frac{1}{2(1+x^2)}$ 1
- $v = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$
 $= 2 \tan^{-1} x$

$$\Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2} \quad 1$$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{4} \quad \frac{1}{2}$$

OR

$$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t \quad \frac{1}{2}$$

$$y = \sin pt \Rightarrow \frac{dy}{dt} = p \cos pt \quad \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{p \cos pt}{\cos t} \quad 1$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{\cos t (-p^2 \sin pt) - p \cos pt (-\sin t)}{\cos^2 t} \cdot \frac{dt}{dx} \\ &= \frac{-p^2 \sin pt \cos t + p \cos pt \sin t}{\cos^3 t} \quad 1 \end{aligned}$$

$$\text{Now } (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0 \left[\text{Substituting values of } y, \frac{dy}{dx} \text{ \& } \frac{d^2y}{dx^2} \right] \quad 1$$

8. Eqn of given curves

$$y^2 = 4ax \text{ and } x^2 = 4by$$

Their point of intersections are (0, 0) and $(4a^{1/3} b^{2/3}, 4a^{2/3} b^{1/3})$ 1

$$y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y}, \text{ slope} = \frac{a^{1/3}}{2b^{1/3}} \quad \dots(i) \quad 1$$

$$x^2 = 4by \Rightarrow \frac{dy}{dx} = \frac{x}{2b}, \text{ slope} = \frac{2a^{1/3}}{b^{1/3}} \quad \dots(ii) \quad 1$$

At (0, 0), angle between two curves is 90°

or

Acute angle θ between (i) and (ii) is

$$\theta = \tan^{-1} \left\{ \frac{3 \left(\frac{a^{1/3} b^{1/3}}{a^{2/3} + b^{2/3}} \right)}{2} \right\} \quad 1$$

$$9. \quad I = \int_0^\pi \frac{(\pi-x)}{1 + \sin \alpha \sin (\pi-x)} dx \quad 1$$

$$2I = \pi \int_0^\pi \frac{dx}{1 + \sin \alpha \sin x}$$

$$= 2\pi \int_0^{\pi/2} \frac{dx}{1 + \sin \alpha \sin x}$$

$$\Rightarrow I = \pi \int_0^{\pi/2} \frac{dx}{1 + \sin \alpha \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} \quad 1$$

$$I = \pi \int_0^1 \frac{2dt}{1+t^2+2t \sin \alpha} \quad \text{Put } \tan \frac{x}{2} = t \quad \frac{1}{2}$$

$$\Rightarrow I = 2\pi \int_0^1 \frac{dt}{(t + \sin \alpha)^2 + \cos^2 \alpha} \quad 1$$

$$= \frac{2\pi}{\cos \alpha} \left[\tan^{-1} \left(\frac{t + \sin \alpha}{\cos \alpha} \right) \right]_0^1$$

$$\Rightarrow I = \frac{\pi}{\cos \alpha} \left(\frac{\pi}{2} - \alpha \right) \quad \frac{1}{2}$$

10. $I = \int (2x+5) \sqrt{10-4x-3x^2} dx$

$$= -\frac{1}{3} \int (-4-6x) \sqrt{10-4x-3x^2} dx + \frac{11}{3} \int \sqrt{10-4x-3x^2} dx \quad 1$$

$$= -\frac{2}{9} (10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3} \int \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x-\frac{2}{3}\right)^2} dx \quad 1+1$$

$$= -\frac{2}{9} (10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3} \left[\frac{\left(x-\frac{2}{3}\right) - \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x-\frac{2}{3}\right)^2}}{2} + \frac{17}{9} \sin^{-1} \frac{3x-2}{\sqrt{34}} \right] + C \quad 1$$

OR

$$x^2 = y \text{ (say)} \quad \frac{1}{2}$$

$$\frac{(y+1)(y+4)}{(y+3)(y-5)} = 1 + \frac{A}{y+3} + \frac{B}{y-5} \quad \frac{1}{2}$$

using partial fraction we get $A = \frac{1}{4}, B = \frac{27}{4}$ 1

$$\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)} dx = \int 1 dx + \frac{1}{4} \int \frac{dx}{x^2+3} + \frac{27}{4} \int \frac{dx}{x^2-5} \quad 1$$

$$= x + \frac{1}{4\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}} \log \left| \frac{x-\sqrt{5}}{x+\sqrt{5}} \right| + C \quad 1$$

11. $I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

$$\text{put } \sin^{-1} x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt \quad \frac{1}{2} + \frac{1}{2}$$

$$= \int t \cdot \sin t dt \quad 1$$

$$= -t \cos t + \sin t + c \quad \frac{1}{2}$$

$$= -\sqrt{1-x^2} \sin^{-1} x + x + c \quad \frac{1}{2}$$

$$12. \quad y^2 dx + (x^2 - xy + y^2) dy = 0$$

$$\Rightarrow \frac{dx}{dy} = -\frac{(x^2 - xy + y^2)}{y^2} \quad \frac{1}{2}$$

$$\text{put } x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy} \quad \frac{1}{2}$$

$$v + y \frac{dv}{dy} = \frac{(v^2 y^2 - y^2 v + y^2)}{y^2}$$

$$\Rightarrow \frac{dv}{v^2 + 1} = -\frac{dy}{y} \quad 1 \frac{1}{2}$$

Integrating both sides

$$\tan^{-1} v = -\log y + c \quad \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \tan^{-1} \frac{x}{y} = -\log y + c \quad \frac{1}{2}$$

$$13. \quad \frac{dx}{dy} - \frac{x}{1+y^2} = \frac{\cot^{-1} y}{1+y^2} \quad \frac{1}{2}$$

$$\text{I.F} = e^{-\int \frac{1}{1+y^2}} = e^{\cot^{-1} y} \quad 1$$

$$\Rightarrow \frac{d}{dy} (x \cdot e^{\cot^{-1} y}) = \frac{\cot^{-1} y \cdot e^{\cot^{-1} y}}{1+y^2}$$

Integrating, we get

$$x \cdot e^{\cot^{-1} y} = \int \frac{\cot^{-1} y \cdot e^{\cot^{-1} y}}{1+y^2} dy \quad 1 \frac{1}{2}$$

put $\cot^{-1} y = t$

$$= -\int t e^t dt$$

$$= (1-t) e^t + c$$

$$\Rightarrow x = (1 - \cot^{-1} y) + c e^{-\cot^{-1} y} \quad 1$$

$$14. \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{d} \quad \dots(i)$$

$$\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \quad \dots(ii)$$

$$(1) - (2) \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{d} \times (\vec{b} - \vec{c}) \quad 2$$

$$\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0} \quad 1$$

$$\Rightarrow (\vec{a} - \vec{d}) \parallel (\vec{b} - \vec{c}) \quad 1$$

15. Equation of line \overline{AB}

$$\vec{r} = (-\hat{j} - \hat{k}) + \lambda (4\hat{i} + 6\hat{j} + 2\hat{k}) \quad 1$$

Equation of line \overline{CD}

$$\vec{r} = (3\hat{i} + 9\hat{j} + 4\hat{k}) + \mu (-7\hat{i} - 5\hat{j}) \quad \frac{1}{2}$$

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 10\hat{j} + 5\hat{k} \quad \frac{1}{2}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 10\hat{i} - 14\hat{j} + 22\hat{k} \quad 1$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 30 - 140 + 110 = 0 \quad 1$$

\Rightarrow Lines intersect

16. Let selection of defective pen be considered success

$$p = \frac{2}{20} = \frac{1}{10}, q = \frac{9}{10} \quad 1$$

$$\text{Reqd probability} = P(x=0) + P(x=1) + P(x=2) \quad 1\frac{1}{2}$$

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 + {}^5C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2} \left(\frac{9}{10}\right)^4 + \frac{1}{10} \times \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^3 \times \frac{34}{25} \quad 1\frac{1}{2}$$

OR

$$\sum_{i=0}^4 P(x_i) = 1 \quad \frac{1}{2}$$

$$\Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8} \quad \frac{1}{2}$$

$$(i) P(x=1) = \frac{1}{8} \quad 1$$

$$(ii) P(\text{at most 2 colleges}) = P(0) + P(1) + P(2)$$

$$= \frac{5}{8} \quad 1$$

$$(iii) P(\text{at least 2 colleges}) = 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \frac{1}{8} = \frac{7}{8} \quad 1$$

$$\begin{aligned}
 17. \quad \text{LHS} &= \cot^{-1} \left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) + \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)}{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) - \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)} \right] && 1+1 \\
 &= \cot^{-1} \left(\cot \frac{x}{2} \right) && 1 \\
 &= \frac{x}{2} = \text{RHS} && 1
 \end{aligned}$$

OR

$$\begin{aligned}
 \tan^{-1} \left[\frac{\frac{x-2}{x-1} + \frac{x+2}{x+1}}{1 - \frac{x-2}{x-1} \cdot \frac{x+2}{x+1}} \right] &= \frac{\pi}{4} && 1\frac{1}{2} \\
 \Rightarrow \frac{2x^2 - 4}{3} &= \tan \frac{\pi}{4} && 1\frac{1}{2} \\
 \Rightarrow x &= \pm \sqrt{\frac{7}{2}} && 1
 \end{aligned}$$

18. Let each poor child pay ₹ x per month and each rich child pay ₹ y per month.

$$\begin{aligned}
 \therefore 20x + 5y &= 9000 && \frac{1}{2} \\
 5x + 25y &= 26000 && \frac{1}{2}
 \end{aligned}$$

In matrix form,

$$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix} \quad 1$$

$$AX = B \Rightarrow X = A^{-1} B$$

$$A^{-1} = \frac{1}{475} \begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475} \begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix} \begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$$\Rightarrow x = 200, y = 1000 \quad 1$$

Value: Compassion or any relevant value 1

19. $f'_{1-} = 2x + 3 = 5$

$$f'_{1+} = b$$

$$f'_{1-} = f'_{1+} \Rightarrow \boxed{b=5} \quad 1+1$$

$$\lim_{x \rightarrow 1^-} f(x) = f(1) = \lim_{x \rightarrow 1^+} f(x)$$

$$\Rightarrow 4 + a = b + 2 \quad 1$$

$$\Rightarrow \boxed{a=3} \quad 1$$

SECTION C

20. Equation of family of planes passing through two given planes

$$(x + 2y + 3z - 4) + k(2x + y - z + 5) = 0 \quad 1$$

$$\Rightarrow (1 + 2k)x + (2 + k)y + (3 - k)z = 4 - 5k \quad \dots(i)$$

$$\Rightarrow \frac{x}{\frac{4-5k}{1+2k}} + \frac{y}{\frac{4-5k}{2+k}} + \frac{z}{\frac{4-5k}{3-k}} = 1$$

As per condition

$$\frac{4-5k}{1+2k} = \frac{2(4-5k)}{(3-k)} \quad 1$$

$$\Rightarrow k = \frac{4}{5} \text{ or } \frac{1}{5} \quad 1$$

For $k = \frac{1}{5}$, Eqn. of plane is $7x + 11y + 14z = 15$ 1

For $k = \frac{4}{5}$, Eqn. of plane is $13x + 14y + 11z = 0$ $\frac{1}{2}$

Equation of plane passing through $(2, 3, -1)$

and parallel to the plane is:

$$7(x-2) + 11(y-3) + 14(z+1) = 0$$

$$\Rightarrow 7x + 11y + 14z = 33 \quad 1$$

Vector form: $\vec{r} \cdot (7\hat{i} + 11\hat{j} + 14\hat{k}) = 33$ $\frac{1}{2}$

21. Let H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 black ball, transferred

H_3 be the event 2 black and 1 black ball transferred

E be the event that ball drawn from B is red. 1

$$P(H_1) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28} \quad P(E/H_1) = \frac{6}{10}$$

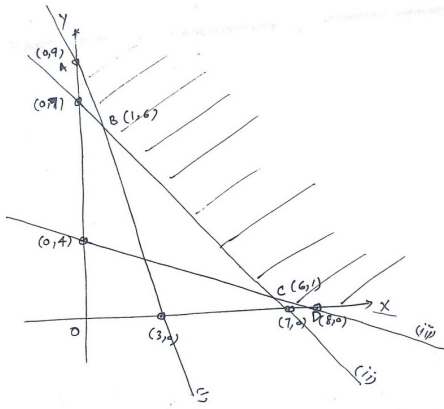
$$P(H_2) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28} \quad P(E/H_2) = \frac{5}{10}$$

$$P(H_3) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28} \quad P(E/H_3) = \frac{4}{10} \quad 1\frac{1}{2} + 1\frac{1}{2}$$

$$P(H_1/E) = \frac{\frac{3}{28} \times \frac{6}{10}}{\frac{3}{28} \times \frac{6}{10} + \frac{15}{28} \times \frac{5}{10} + \frac{10}{28} \times \frac{4}{10}} \quad 1\frac{1}{2}$$

$$= \frac{18}{133} \quad 1\frac{1}{2}$$

22.



Let x tablets of type X and y tablets of type Y are taken

Minimise $C = 2x + y$

subjected to

$$\left. \begin{aligned} 6x + 2y &\geq 18 \\ 3x + 3y &\geq 21 \\ 2x + 4y &\geq 16 \\ x, y &\geq 0 \end{aligned} \right\}$$

Correct Graph

$Cl_{A(0,9)} = 9$

$Cl_{B(1,6)} = 8 \leftarrow$ Minimum value

$Cl_{C(6,1)} = 13$

$Cl_{D(8,0)} = 16$

$2x + y < 8$ does not pass through unbounded region

Thus, minimum value of $C = 8$ at $x = 1, y = 6$.

23. $f(x) = |x| + x, \quad g(x) = |x| - x \quad \forall x \in \mathbb{R}$

$(f \circ g)(x) = f(g(x))$

$= ||x| - 1| + |x| - x$

$(g \circ f)(x) = g(f(x))$

$= ||x| + x| - |x| - x$

$(f \circ g)(-3) = 6$

$(f \circ g)(5) = 0$

$(g \circ f)(-2) = 2$

24. $abc \begin{vmatrix} \frac{1}{a} + 1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} + 1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0$

$C_1 \rightarrow C_1 + C_2 + C_3$

$\Rightarrow abc \begin{vmatrix} 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} + 1 & \frac{1}{c} \\ 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0$

$$\Rightarrow abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b} + 1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1 \quad 1\frac{1}{2}$$

$$\Rightarrow abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = 0$$

$$\therefore a, b, c, \neq 0$$

$$\therefore 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \quad 1\frac{1}{2}$$

OR

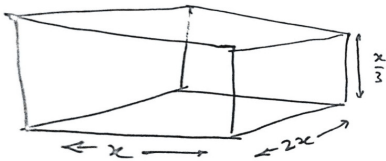
$$|A| = 1 \quad 1$$

$$\text{adj } A = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 3$$

$$A(\text{adj } A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad 1$$

$$|A| I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad 1$$

25.



$$S = 6x^2 + 4\pi r^2$$

$$\Rightarrow r = \sqrt{\frac{S - 6x^2}{4\pi}} \quad \dots(i) \quad 1\frac{1}{2}$$

$$V = \frac{2x^3}{3} + \frac{4}{3}\pi r^3 \quad 1$$

$$= \frac{2}{3}x^3 + \frac{4}{3}\pi \left(\frac{S - 6x^2}{4\pi} \right)^{3/2}$$

$$= \frac{2x^3}{3} + \frac{(S - 6x^2)^{3/2}}{6\sqrt{\pi}} \quad 1$$

$$\frac{dV}{dx} = 2x^2 - \frac{3x}{\sqrt{\pi}} \sqrt{S - 6x^2} \quad 1$$

$$\frac{dV}{dx} = 0 \Rightarrow 2x^2 = \frac{3x \sqrt{S - 6x^2}}{\sqrt{\pi}}$$

$$\Rightarrow r = \frac{x}{3} \text{ [using (i)]} \quad 1$$

$$\frac{d^2V}{dx^2} = 4x \left[\frac{3x}{\sqrt{\pi}} \cdot \frac{(-12x)}{2\sqrt{S-6x^2}} + \frac{3}{\sqrt{\pi}} \sqrt{S-6x^2} \right]$$

1

$$\left. \frac{d^2V}{dx^2} \right|_{x=\frac{r}{3}} > 0$$

$\Rightarrow V$ is minimum at $x = \frac{r}{3}$ i.e. $r = 3x$

$$\text{Minimum value of sum of volume} = \left(\frac{2x^3}{3} + 36\pi x^3 \right) \text{ cubic units} \quad \frac{1}{2}$$

OR

Equation of given curve

$$y = \cos(x + y) \quad \dots(i)$$

$$\Rightarrow \frac{dy}{dx} = -\sin(x + y) \left(1 + \frac{dy}{dx} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x + y)}{1 + \sin(x + y)} \quad 2$$

$$\text{given line } x + 2y = 0, \text{ its slope} = -\frac{1}{2} \quad \frac{1}{2}$$

condition of || lines

$$\frac{-\sin(x + y)}{1 + \sin(x + y)} = -\frac{1}{2} \quad 1$$

$$\Rightarrow \sin(x + y) = 1$$

$$\Rightarrow \cos(x + y) = 0 \quad y = 0 \quad \text{using (i)}$$

$$\Rightarrow \cos x = 0 \Rightarrow x = (2n + 1) \frac{\pi}{2}, n \in \mathbb{I}$$

$$\therefore x = \frac{-3\pi}{2}, \frac{\pi}{2} \in [-2\pi, 2\pi] \quad 1$$

Thus tangents are || to the line $x + 2y = 0$

$$\text{only at pts } \left(-\frac{3\pi}{2}, 0 \right) \text{ and } \left(\frac{\pi}{2}, 0 \right) \quad \frac{1}{2}$$

\therefore Required equation of tangents are

$$y - 0 = -\frac{1}{2} \left(x + \frac{3\pi}{2} \right) \Rightarrow 2x + 4y + 3\pi = 0 \quad \frac{1}{2}$$

$$y - 0 = -\frac{1}{2} \left(x - \frac{\pi}{2} \right) \Rightarrow 2x + 4y - \pi = 0 \quad \frac{1}{2}$$

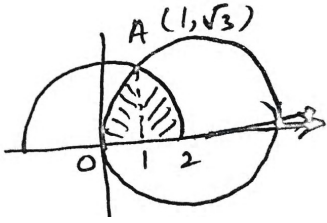
26.

Their point of intersection $(1, \sqrt{3})$

1

Correct Figure

1



$$\text{Required Area} = \int_0^1 \sqrt{(2)^2 - (x-2)^2} dx + \int_1^2 \sqrt{2^2 - x^2} dx$$

2

$$= \left[\frac{(x-2)\sqrt{4x-x^2}}{2} + 2 \sin^{-1} \frac{x-2}{2} \right]_0^1 + \left[\frac{x\sqrt{4-x^2}}{2} + 2 \sin^{-1} \frac{x}{2} \right]_1^2$$

1

$$= \left(\frac{5\pi}{3} - \sqrt{3} \right) \text{Sq. units}$$

1