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HALF YEARLY EXAMINATION, 2017-18

MATHEMATICS

Time : 3 hrs. **Class – XII** **M.M. : 80**

General Instructions :

(1) All questions are compulsory.

(2) This question paper contains 29 questions.

(3) Questions 1-4 in Section A are very short-answer type questions carrying 1 mark each.

(4) Questions 5-12 in Section B are short-answer type questions carrying 2 marks each.

(5) Questions 13-23 in Section C are long-answer I type questions carrying 4 marks each.

(6) Questions 24-29 in Section D are long-answer II type questions carrying 6 marks each.

Section—A

1 Construct a 2×2 matrix whose elements are given by :

$a_{ij} = |2i - j|$

2 If A is a square matrix of order 3 such that $|\text{adj } A| = 144$, write the value of $|A|$.

3 Evaluate :

$$\int \frac{1+\cos x}{x+\sin x}\,dx$$

4 Write sum of the order and degree of the differential equation :

$$(a+x)\frac{d^2y}{dx^2}+y=0$$

Section—B

5 If $f : R \to R$ is defined by $f(x) = 2x + 3$, write the value of $f(f(x))$:

6 Evaluate :

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \bullet \cos^2 x}\,dx$$

7 Prove that the relation on the set Z of all integers defined by $(a, b) \in R \Leftrightarrow a-b$ is divisible by 5 is an equivalence relation.

8 Find the value of λ, such that $2\bar{i}-\lambda\bar{j}+\bar{k}$ and $\bar{i}+\bar{j}+\bar{k}$ are perpendicular.

9 If $x^x = y^y$, then find $\frac{dy}{dx}$.

10 Let A = (1, 2, 3) and B = (2, –3, 5). Find the direction ratios of AB. Also find scaler & vector component of vector AB.

11 Show that the function $f : R \to R$ defined by $f(x)=\frac{2x-1}{3}$, $x \in R$, is one-one and onto function. Also find the inverse of the function f.

12 Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

Section—C

13 Using properties of determinants show that :

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

14 Find all the points of discontinuity of the function f defined by :

$$f(x)\begin{cases} x+2 & x \le 1 \\ x-2 & 1 < x < 2 \\ 0 & x \ge 2 \end{cases}$$

15 If x = a(cos t + t sin t) and y = a(sin t – t cos t), $0 < t < \frac{\pi}{2}$, find $\frac{d^2y}{dx^2}$.

OR

If $y = \sqrt{\frac{1-x}{1+x}}$, show that $(1-x)\frac{dy}{dx} + y = 0$.

16 Find the equation of the normal to the curve $y = x^3 + 2x + 6$ at point (0, 6).

17 Evaluate :

$$\int \frac{3x+2}{(x-1)(2x+3)}\,dx$$

18 Prove that :

$$2\tan^{-1}\frac{1}{5} + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

19 Solve the following differential equation :

$$\frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{1}{(x^2+1)^2} \; ; \; y(0) = 0$$

OR

Solve the following differential equation :

$(x^3 + y^3)dy - x^2y\,dx = 0$

20 Show that the vectors :

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k},\ \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ *and* $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

OR

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k},\ \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ *and* $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 5$.

21 Differentiate :

$$\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) w.r.t.x$$

22 Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence, find A^{-1}.

23 Evaluate :

$$\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$$

Section—C

24 Using matrix method, solve the following system of linear equations :

$x + 2y + z = 4,\ x - y - z = 0,\ x - 3y + z = 2$

OR

Find the inverse of the following matrix by using elementary row transformation method

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & -1 \\ 1 & -3 & 1 \end{bmatrix}$$

25 A wire of length 36 cm is cut into two pieces is turned in the form of a circle and the other in the form of an equilateral triangle. Find the length of each piece so that the sum of area of the two be minimum.

OR

Show that the right circular cylinder of given volume open at the top has minimum total surface area, provided its height is equal to the radius of its base.

26 Evaluate the following integral as limit of sums :

$$\int_1^4 (x^2 - x)dx$$

27 Using integration, find area of whose vertices have coordinates :

A(2, 0), B(4, 5) and C(6, 3)

28 Let A = Z × Z and * be a binary operation on A defined by :

(a, b) * (c, d) = (a + c , b + d),

(a) Examine whether '*' is commutative and associative.

(b) Find the identity element in A.

(c) Find the invertible elements of A.

29 A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is ₹ 30 while by selling one table the profit is ₹ 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit ? Formulate the problem as L.P.P. and solve it graphically.