

Chapter Four

MOTION IN A PLANE



MCQ I

4.1 The angle between $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$ is

- (a) 45° (b) 90° (c) -45° (d) 180°

4.2 Which one of the following statements is true?

- (a) A scalar quantity is the one that is conserved in a process.
(b) A scalar quantity is the one that can never take negative values.
(c) A scalar quantity is the one that does not vary from one point to another in space.
(d) A scalar quantity has the same value for observers with different orientations of the axes.

4.3 Figure 4.1 shows the orientation of two vectors \mathbf{u} and \mathbf{v} in the XY plane.

If $\mathbf{u} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and

$\mathbf{v} = p\hat{\mathbf{i}} + q\hat{\mathbf{j}}$

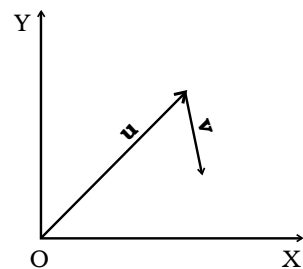


Fig. 4.1

- which of the following is correct?
- (a) a and p are positive while b and q are negative.
 - (b) a , p and b are positive while q is negative.
 - (c) a , q and b are positive while p is negative.
 - (d) a , b , p and q are all positive.
- 4.4** The component of a vector \mathbf{r} along X -axis will have maximum value if
- (a) \mathbf{r} is along positive Y -axis
 - (b) \mathbf{r} is along positive X -axis
 - (c) \mathbf{r} makes an angle of 45° with the X -axis
 - (d) \mathbf{r} is along negative Y -axis
- 4.5** The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45° , its range will be
- (a) 60 m
 - (b) 71 m
 - (c) 100 m
 - (d) 141 m
- 4.6** Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are
- (a) Impulse, pressure and area
 - (b) Impulse and area
 - (c) Area and gravitational potential
 - (d) Impulse and pressure
- 4.7** In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following are necessarily true?
- (a) The average velocity is not zero at any time.
 - (b) Average acceleration must always vanish.
 - (c) Displacements in equal time intervals are equal.
 - (d) Equal path lengths are traversed in equal intervals.
- 4.8** In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following are necessarily true?
- (a) The acceleration of the particle is zero.
 - (b) The acceleration of the particle is bounded.
 - (c) The acceleration of the particle is necessarily in the plane of motion.
 - (d) The particle must be undergoing a uniform circular motion

4.9 Three vectors \mathbf{A}, \mathbf{B} and \mathbf{C} add up to zero. Find which is false.

- (a) $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$ is not zero unless \mathbf{B}, \mathbf{C} are parallel
- (b) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ is not zero unless \mathbf{B}, \mathbf{C} are parallel
- (c) If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ define a plane, $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$ is in that plane
- (d) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = |\mathbf{A}| |\mathbf{B}| |\mathbf{C}| \rightarrow C^2 = A^2 + B^2$

4.10 It is found that $|\mathbf{A} + \mathbf{B}| = |\mathbf{A}|$. This necessarily implies,

- (a) $\mathbf{B} = \mathbf{0}$
- (b) \mathbf{A}, \mathbf{B} are antiparallel
- (c) \mathbf{A}, \mathbf{B} are perpendicular
- (d) $\mathbf{A} \cdot \mathbf{B} \leq 0$

MCQ II

4.11 Two particles are projected in air with speed v_0 at angles θ_1 and θ_2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then tick the right choices

- (a) angle of projection : $q_1 > q_2$
- (b) time of flight : $T_1 > T_2$
- (c) horizontal range : $R_1 > R_2$
- (d) total energy : $U_1 > U_2$.

4.12 A particle slides down a frictionless parabolic ($y = x^2$) track (A – B – C) starting from rest at point A (Fig. 4.2). Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then

- (a) KE at P = KE at B
- (b) height at P = height at A
- (c) total energy at P = total energy at A
- (d) time of travel from A to B = time of travel from B to P.

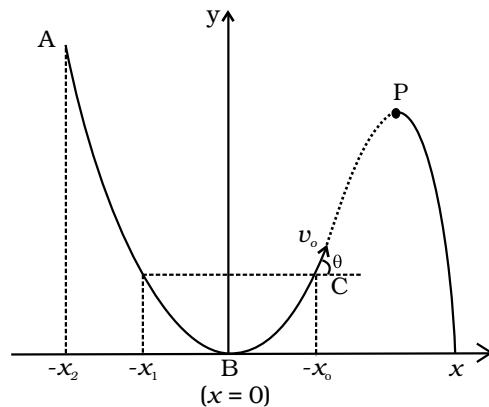


Fig. 4.2

4.13 Following are four different relations about displacement, velocity and acceleration for the motion of a particle in general. Choose the incorrect one (s) :

- (a) $\mathbf{v}_{av} = \frac{1}{2} [\mathbf{v}(t_1) + \mathbf{v}(t_2)]$
- (b) $\mathbf{v}_{av} = \frac{\mathbf{r}(t_2) - \mathbf{r}(t_1)}{t_2 - t_1}$

$$(c) \mathbf{r} = \frac{1}{2}(\mathbf{v}(t_2) - \mathbf{v}(t_1))(t_2 - t_1)$$

$$(d) \mathbf{a}_{av} = \frac{\mathbf{v}(t_2) - \mathbf{v}(t_1)}{t_2 - t_1}$$

4.14 For a particle performing uniform circular motion, choose the correct statement(s) from the following:

- (a) Magnitude of particle velocity (speed) remains constant.
- (b) Particle velocity remains directed perpendicular to radius vector.
- (c) Direction of acceleration keeps changing as particle moves.
- (d) Angular momentum is constant in magnitude but direction keeps changing.

4.15 For two vectors \mathbf{A} and \mathbf{B} , $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$ is always true when

- (a) $|\mathbf{A}| = |\mathbf{B}| \neq 0$
- (b) $\mathbf{A} \perp \mathbf{B}$
- (c) $|\mathbf{A}| = |\mathbf{B}| \neq 0$ and \mathbf{A} and \mathbf{B} are parallel or anti parallel
- (d) when either $|\mathbf{A}|$ or $|\mathbf{B}|$ is zero.

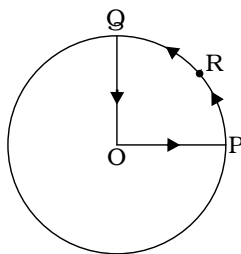


Fig. 4.3

VSA

4.16 A cyclist starts from centre O of a circular park of radius 1km and moves along the path OPRQO as shown Fig. 4.3. If he maintains constant speed of 10ms^{-1} , what is his acceleration at point R in magnitude and direction?

4.17 A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Fig. 4.4, where x and y indicate horizontal and vertical directions, respectively. Show in the diagram, direction of velocity and acceleration at points A, B and C.

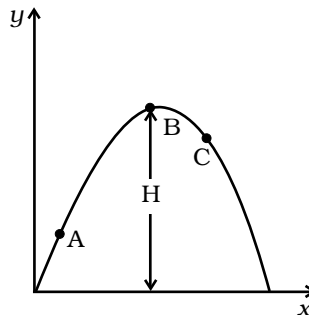


Fig. 4.4

- 4.18** A ball is thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have
- (a) greatest speed.
 - (b) smallest speed.
 - (c) greatest acceleration?
- Explain
- 4.19** A football is kicked into the air vertically upwards. What is its
- (a) acceleration, and (b) velocity at the highest point?
- 4.20** **A**, **B** and **C** are three non-collinear, non co-planar vectors. What can you say about direction of $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$?

SA

- 4.21** A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.
- 4.22** A boy throws a ball in air at 60° to the horizontal along a road with a speed of 10 m/s (36km/h). Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18km/h). Give explanation to support your diagram.
- 4.23** In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included? Sketch such a trajectory and explain why you have drawn it that way.
- 4.24** A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/h. At what angle of sight (w.r.t. horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target?
- 4.25** (a) Earth can be thought of as a sphere of radius 6400 km. Any object (or a person) is performing circular motion around the axis of earth due to earth's rotation (period 1 day). What is acceleration of object on the surface of the earth (at equator) towards its centre? what is it at latitude θ ? How does these accelerations compare with $g = 9.8 \text{ m/s}^2$?

- (b) Earth also moves in circular orbit around sun once every year with an orbital radius of $1.5 \times 10^{11} \text{ m}$. What is the acceleration of earth (or any object on the surface of the earth) towards the centre of the sun? How does this acceleration compare with $g = 9.8 \text{ m/s}^2$?

(Hint : acceleration $\frac{V^2}{R} = \frac{4\pi^2 R}{T^2}$)

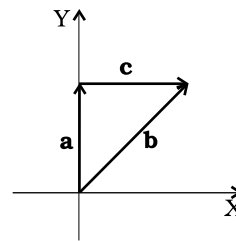
4.26 Given below in column I are the relations between vectors **a**, **b** and **c** and in column II are the orientations of **a**, **b** and **c** in the XY plane. Match the relation in column I to correct orientations in column II.

Column I

Column II

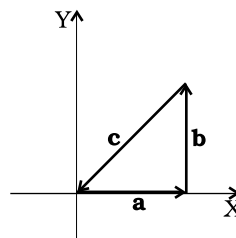
(a) $\mathbf{a} + \mathbf{b} = \mathbf{c}$

(i)



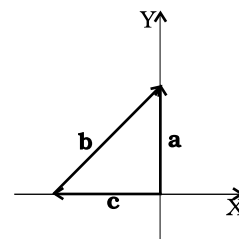
(b) $\mathbf{a} - \mathbf{c} = \mathbf{b}$

(ii)



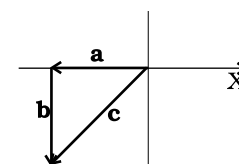
(c) $\mathbf{b} - \mathbf{a} = \mathbf{c}$

(iii)



(d) $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$

(iv)



4.27 If $|\mathbf{A}| = 2$ and $|\mathbf{B}| = 4$, then match the relations in column I with the angle θ between \mathbf{A} and \mathbf{B} in column II.

Column I	Column II
(a) $\mathbf{A} \cdot \mathbf{B} = 0$	(i) $\theta = 0$
(b) $\mathbf{A} \cdot \mathbf{B} = +8$	(ii) $\theta = 90^\circ$
(c) $\mathbf{A} \cdot \mathbf{B} = 4$	(iii) $\theta = 180^\circ$
(d) $\mathbf{A} \cdot \mathbf{B} = -8$	(iv) $\theta = 60^\circ$

4.28 If $|\mathbf{A}| = 2$ and $|\mathbf{B}| = 4$, then match the relations in column I with the angle θ between \mathbf{A} and \mathbf{B} in column II

Column I	Column II
(a) $ \mathbf{A} \times \mathbf{B} = 0$	(i) $\theta = 30^\circ$
(b) $ \mathbf{A} \times \mathbf{B} = 8$	(ii) $\theta = 45^\circ$
(c) $ \mathbf{A} \times \mathbf{B} = 4$	(iii) $\theta = 90^\circ$
(d) $ \mathbf{A} \times \mathbf{B} = 4\sqrt{2}$	(iv) $\theta = 0^\circ$

LA

4.29 A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800m from the foot of hill and can be moved on the ground at a speed of 2 m/s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take $g = 10 \text{ m/s}^2$.

4.30 A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{g}$.

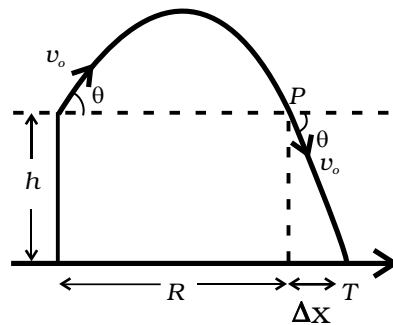


Fig 4.5

If a target farther away by distance Δx (beyond R) has to be hit with the same gun (Fig 4.5), show that it could be achieved by raising the gun to a height at least

$$h = \Delta x \left[1 + \frac{\Delta x}{R} \right]$$

(Hint : This problem can be approached in two different ways:

- (i) Refer to the diagram: target T is at horizontal distance $x = R + \Delta x$ and below point of projection $y = -h$.
- (ii) From point P in the diagram: Projection at speed v_0 at an angle θ below horizontal with height h and horizontal range Δx .)

4.31 A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal (Fig. 4.6).

- (a) Find an expression of range on the plane surface (distance on the plane from the point of projection at which particle will hit the surface).
- (b) Time of flight.
- (c) β at which range will be maximum.

Fig. 4.6

(Hint : This problem can be solved in two different ways:

- (i) Point P at which particle hits the plane can be seen as intersection of its trajectory (parabola) and straight line. Remember particle is projected at an angle $(\alpha + \beta)$ w.r.t. horizontal.
- (ii) We can take x -direction along the plane and y -direction perpendicular to the plane. In that case resolve g (acceleration due to gravity) in two different components, g_x along the plane and g_y perpendicular to the plane. Now the problem can be solved as two independent motions in x and y directions respectively with time as a common parameter.)

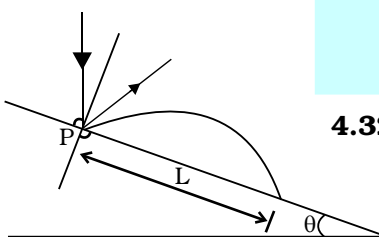


Fig 4.7

4.32 A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v_0 and rebounds elastically (Fig 4.7). Find the distance along the plane where it will hit second time.

- (Hint: (i) After rebound, particle still has speed V_0 to start.
 (ii) Work out angle particle speed has with horizontal after it rebounds.
 (iii) Rest is similar to if particle is projected up the incline.)

4.33 A girl riding a bicycle with a speed of 5 m/s towards north direction, observes rain falling vertically down. If she increases her speed to 10 m/s, rain appears to meet her at 45° to the vertical. What is the speed of the rain? In what direction does rain fall as observed by a ground based observer?

(Hint: Assume north to be \hat{i} direction and vertically downward to be $-\hat{j}$. Let the rain velocity \mathbf{v}_r be $a\hat{i}+b\hat{j}$. The velocity of rain as observed by the girl is always $\mathbf{v}_r - \mathbf{v}_{girl}$. Draw the vector diagram/s for the information given and find a and b . You may draw all vectors in the reference frame of ground based observer.)

4.34 A river is flowing due east with a speed 3m/s. A swimmer can swim in still water at a speed of 4 m/s (Fig. 4.8).

- If swimmer starts swimming due north, what will be his resultant velocity (magnitude and direction)?
- If he wants to start from point A on south bank and reach opposite point B on north bank,
 - which direction should he swim?
 - what will be his resultant speed?
- From two different cases as mentioned in (a) and (b) above, in which case will he reach opposite bank in shorter time?

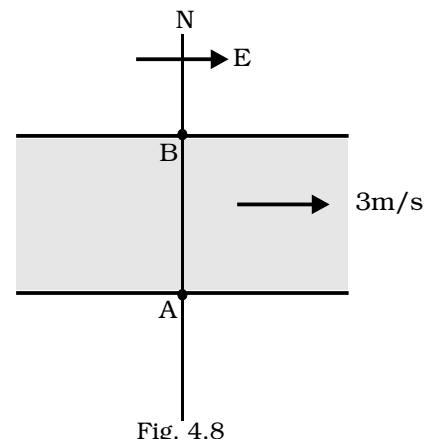


Fig. 4.8

4.35 A cricket fielder can throw the cricket ball with a speed v_0 . If he throws the ball while running with speed u at an angle θ to the horizontal, find

- the effective angle to the horizontal at which the ball is projected in air as seen by a spectator.
- what will be time of flight?
- what is the distance (horizontal range) from the point of projection at which the ball will land?

- (d) find θ at which he should throw the ball that would maximise the horizontal range as found in (iii).
- (e) how does θ for maximum range change if $u > v_0$, $u = v_0$, $u < v_0$?
- (f) how does θ in (v) compare with that for $u = 0$ (i.e. 45°)?

4.36 Motion in two dimensions, in a plane can be studied by expressing position, velocity and acceleration as vectors in Cartesian co-ordinates $\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$ where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vector along x and y directions, respectively and A_x and A_y are corresponding components of \mathbf{A} (Fig. 4.9). Motion can also be studied by expressing vectors in circular polar co-ordinates as $\mathbf{A} = A_r \hat{\mathbf{r}} + A_\theta \hat{\boldsymbol{\theta}}$

where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \cos \theta \hat{\mathbf{i}} + \sin \theta \hat{\mathbf{j}}$ and $\hat{\boldsymbol{\theta}} = -\sin \theta \hat{\mathbf{i}} + \cos \theta \hat{\mathbf{j}}$ are unit vectors along direction in which 'r' and ' θ ' are increasing.

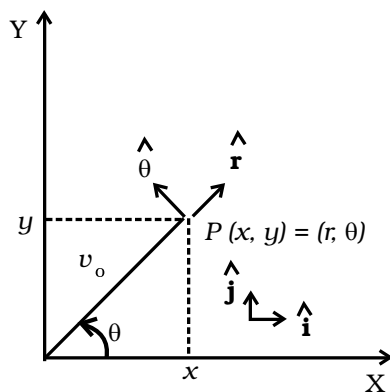


Fig. 4.9

- (a) Express $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ in terms of $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$
- (b) Show that both $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors and are perpendicular to each other.
- (c) Show that $\frac{d}{dt}(\hat{\mathbf{r}}) = \omega \hat{\boldsymbol{\theta}}$ where

$$\omega = \frac{d\theta}{dt} \text{ and } \frac{d}{dt}(\hat{\boldsymbol{\theta}}) = -\omega \hat{\mathbf{r}}$$

- (d) For a particle moving along a spiral given by $\mathbf{r} = a\theta \hat{\mathbf{r}}$, where $a = 1$ (unit), find dimensions of 'a'.
- (e) Find velocity and acceleration in polar vector representation for particle moving along spiral described in (d) above.

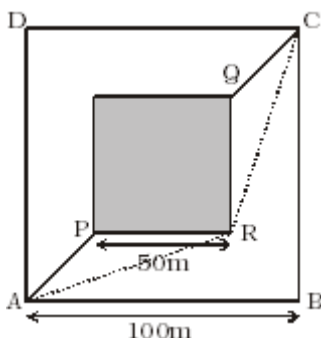


Fig. 4.10

4.37 A man wants to reach from A to the opposite corner of the square C (Fig. 4.10). The sides of the square are 100 m. A central square of 50m \times 50m is filled with sand. Outside this square, he can walk at a speed 1 m/s. In the central square, he can walk only at a speed of v m/s ($v < 1$). What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?