Chapter 12

INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

12.1 Overview

12.1.1 ***Coordinate axes and coordinate planes*** Let X′OX, Y′OY, Z′OZ be three mutually perpendicular lines that pass through a point O such that X′OX and Y′OY lies in the plane of the paper and line Z′OZ is perpendicular to the plane of paper. These three lines are called rectangular axes (lines X′OX, Y′OY and Z′OZ are called x-axis, y-axis and z-axis). We call this coordinate system a three dimensional space, or simply space.

The three axes taken together in pairs determine xy, yz, zx-plane , i.e., three coordinate planes. Each plane divide the space in two parts and the three coordinate planes together divide the space into eight regions (parts) called octant, namely (i) OXYZ (ii) OX′YZ (iii) OXY′Z (iv) OXYZ′ (v) OXY′Z′ (vi) OX′YZ′ (vii) OX′Y′Z (viii) OX′Y′Z′. (Fig.12.1).

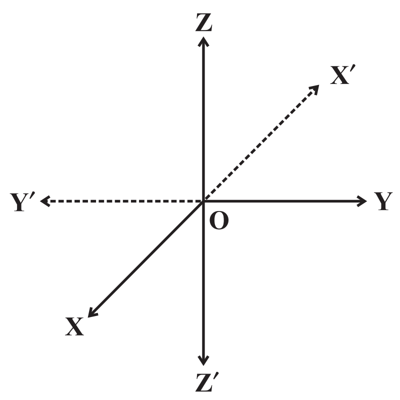

Fig. 12.1

Let P be any point in the space, not in a coordinate plane, and through P pass planes parallel to the coordinate planes yz, zx and xy meeting the coordinate axes in the points A, B, C respectively.

Three planes are

(i) ADPF || yz-plane (ii) BDPE || xz-plane

(iii) CFPE || xy-plane

These planes determine a rectangular parallelopiped which has three pairs of rectangular faces

(A D P F, O B E C),(B D P E, C F A O) and (A O B D, FPEC) (Fig 12.2)

12.1.2 ***Coordinate of a point in space*** An arbitrary point P in three-dimensional space is assigned coordinates (x_0, y_0, z_0) provided that

(1) the plane through P parallel to the *yz*-plane intersects the *x*-axis at $(x_0, 0, 0)$;

(2) the plane through P parallel to the *xz*-plane intersects the *y*-axis at $(0, y_0, 0)$;

(3) the plane through P parallel to the *xy*-plane intersects the *z*-axis at $(0, 0, z_0)$.

The space coordinates (x_0, y_0, z_0) are called the Cartesian coordinates of P or simply the rectangular coordinates of P.

Moreover we can say, the plane ADPF (Fig.12.2) is perpendicular to the *x*-axis or *x*-axis is perpendicular to the plane ADPF and hence perpendicular to every line in the plane. Therefore, PA is perpendicular to OX and OX is perpendicular to PA. Thus A is the foot of perpendicular drawn from P on *x*-axis and distance of this foot A from O is *x*-coordinate of point P. Similarly, we call B and C are the feet of perpendiculars drawn from point P on the *y* and *z*-axis and distances of these feet B and C from O are the *y* and *z* coordinates of the point P.

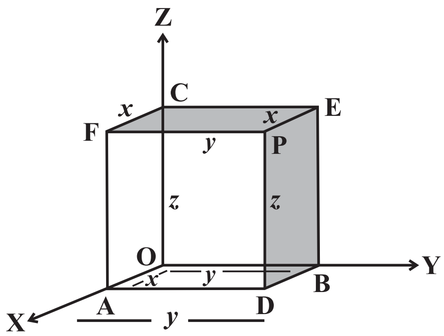

Fig. 12.2

Hence the coordinates *x*, *y* *z* of a point P are the perpendicular distance of P from the three coordinate planes *yz*, *zx* and *xy*, respectively.

12.1.3 ***Sign of coordinates of a point*** The distance measured along or parallel to OX, OY, OZ will be positive and distance moved along or parallel to OX′, OY′, OZ′ will be negative. The three mutually perpendicular coordinate plane which in turn divide the space into eight parts and each part is know as octant. The sign of the coordinates of a point depend upon the octant in which it lies. In first octant all the coordinates are positive and in seventh octant all coordinates are negative. In third octant *x*, *y* coordinates are negative and *z* is positive. In fifth octant *x*, *y* are positive and *z* is negative. In fourth octant *x*, *z* are positive and *y* is negative. In sixth octant *x*, *z* are negative *y* is positive. In the second octant *x* is negative and *y* and *z* are positive.

Octants → Coordinates ↓	I OXYZ	II OX′YZ	III OX′Y′Z	IV OXY′Z	V OXYZ′	VI OX′YZ′	VII OX′Y′Z′	VIII OXY′Z′
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

12.1.4 *Distance formula* The distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

A paralleopiped is formed by planes drawn through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) parallel to the coordinate planes. The length of edges are $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$ and length of diagonal is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

12.1.5 *Section formula* The coordinates of the point R which divides the line segment joining two points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) internally or externally in the ratio $m : n$ are given by $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right), \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$, respectively.

The coordinates of the mid-point of the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and x_3, y_3, z_3 are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$.

12.2 Solved Examples

Short Answer Type

Example 1 Locate the points (i) (2, 3, 4) (ii) (–2, –2, 3) in space.

Solution

(i) To locate the point (2, 3, 4) in space, we move 2 units from O along the positive direction of x-axis. Let this point be A (2, 0, 0). From the point A moves 3 units parallel to +ve direction of y-axis.Let this point be B (2, 3, 0). From the point B moves 4 units along positive direction of z-axis. Let this point be P (2, 3, 4) Fig.(12.3).

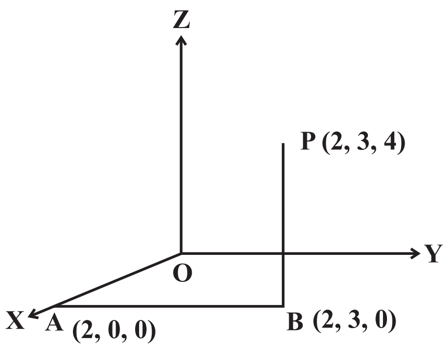

Fig. 12.3

(ii) From the origin, move 2 units along the negative direction of x-axis. Let this point be A (–2, 0, 0). From the point A move 2 units parallel to negative direction of y-axis.

Let this point be B (–2, –2, 0). From B move 3 units parallel to positive direction of z - axis. This is our required point Q (–2, –2, 3) (Fig.12.4.)

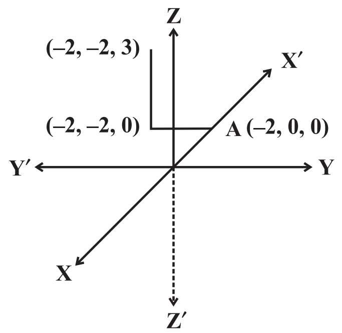

Fig. 12.4

Example 2 Sketch the plane (i) $x = 1$ (ii) $y = 3$ (iii) $z = 4$

Solution

(i) The equation of the plan $x = 0$ represents the yz-plane and equation of the plane $x = 1$ represents the plane parallel to yz-plane at a distance 1 unit above yz-plane. Now, we draw a plane parallel to yz- plane at a distance 1 unit above yz-plane Fig.12.5(a).

(ii) The equation of the plane $y = 0$ represents the xz plane and the equation of the plane $y = 3$ represents the plane parallel to xz plane at a distance 3 unit above xz plane (Fig. 12.5(b)).

(iii) The equation of the plane $z = 0$ represents the xy-plane and $z = 3$ represents the plane parallel to xy-plane at a distance 3 unit above xy-plane (Fig. 12.5(c)).

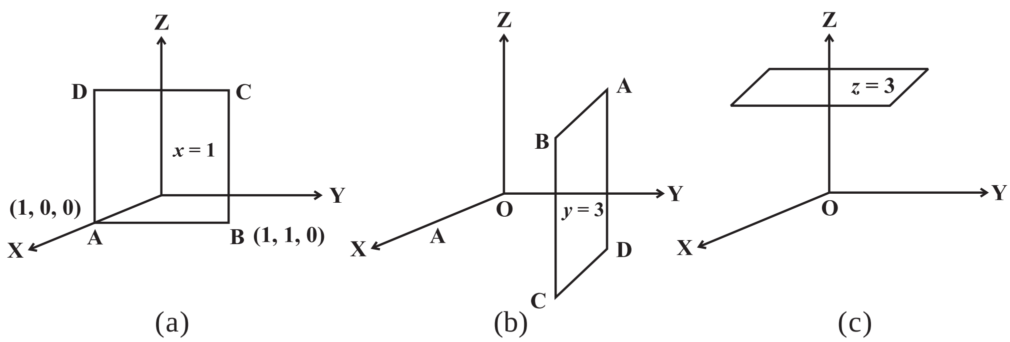

Fig. 12.5

Example 3 Let L, M, N be the feet of the perpendiculars drawn from a point P (3, 4, 5) on the *x*, *y* and *z*-axes respectively. Find the coordinates of L, M and N.

Solution Since L is the foot of perpendicular from P on the *x*-axis, its *y* and *z* co-ordinates are zero. The coordinates of L is (3, 0, 0). Similarly, the coordinates of M and N are (0, 4, 0) and (0, 0, 5), respectively.

Example 4 Let L, M, N be the feet of the perpendicular segments drawn from a point P (3, 4, 5) on the *xy*, *yz* and *zx*-planes, respectively. What are the coordinates of L, M and N?

Solution Since L is the foot of perpendicular segment from P on the *xy*-plane, *z*-coordinate is zero in the *xy*-plane. Hence, coordinates of L is (3, 4, 0). Similarly, we can find the coordinates of of M (0, 4, 5) and N (3, 0, 5), Fig.12.6.

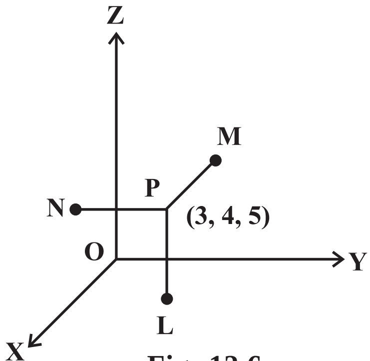

Fig. 12.6

Example 5 Let L, M, N are the feet of the perpendiculars drawn from the point P (3, 4, 5) on the *xy*, *yz* and *zx*-planes, respectively. Find the distance of these points L, M, N from the point P, Fig.12.7.

Solution L is the foot of perpendicular drawn from the point P (3, 4, 5) to the *xy*-plane. Therefore, the coordinate of the point L is (3, 4, 0). The distance between the point (3, 4, 5) and (3, 4, 0) is 5. Similarly, we can find the lengths of the foot of perpendiculars on *yz* and *zx*-plane which are 3 and 4 units, respectively.

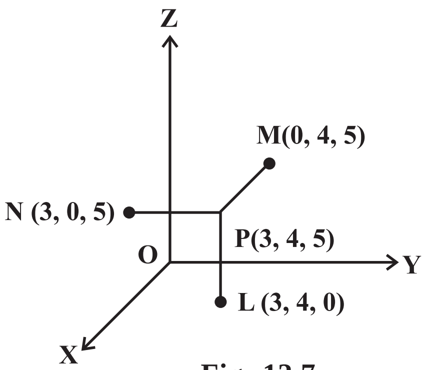

Fig. 12.7

Example 6 Using distance formula show that the points P (2, 4, 6), Q (– 2, – 2, – 2) and R (6, 10, 14) are collinear.

Solution Three points are collinear if the sum of any two distances is equal to the third distance.

$$PQ = \sqrt{(-2-2)^2+(-2-4)^2+(-2-6)^2} = \sqrt{16+36+64} = \sqrt{116} = 2\sqrt{29}$$

$$QR = \sqrt{(6+2)^2+(10+2)^2+(14+2)^2} = \sqrt{64+144+256} = \sqrt{464} = 4\sqrt{29}$$

$$PR = \sqrt{(6-2)^2+(10-4)^2+(14-6)^2} = \sqrt{16+36+64} = \sqrt{116} = 2\sqrt{29}$$

Since QR = PQ + PR. Therefore, the given points are collinear.

Example 7 Find the coordinates of a point equidistant from the four points O (0, 0, 0), A (l, 0, 0), B (0, m, 0) and C (0, 0, n).

Solution Let P (x, y, z) be the required point. Then OP = PA = PB = PC.

Now OP = PA $\Rightarrow$ OP^2 = PA^2 $\Rightarrow x^2 + y^2 + z^2 = (x - l)^2 + (y - 0)^2 + (z - 0)^2 \Rightarrow x = \frac{l}{2}$

Similarly, OP = PB $\Rightarrow y = \frac{m}{2}$ and OP = PC $\Rightarrow z = \frac{n}{2}$

Hence, the coordinate of the required point are $\left(\frac{l}{2}, \frac{m}{2}, \frac{n}{2}\right)$.

Example 8 Find the point on x-axis which is equidistant from the point A (3, 2, 2) and B (5, 5, 4).

Solution The point on the x-axis is of form P (x, 0, 0). Since the points A and B are equidistant from P. Therefore PA^2 = PB^2, i.e.,

$(x - 3)^2 + (0 - 2)^2 + (0 - 2)^2 = (x - 5)^2 + (0 - 5)^2 + (0 - 4)^2$

$\Rightarrow 4x = 25 + 25 + 16 - 17$ i.e., $x = \frac{49}{4}$.

Thus, the point P on the x - axis is $\left(\frac{49}{4}, 0, 0\right)$ which is equidistant from A and B.

Example 9 Find the point on y-axis which is at a distance $\sqrt{10}$ from the point (1, 2, 3)

Solution Let the point P be on y-axis. Therefore, it is of the form P (0, y, 0).

The point (1, 2, 3) is at a distance $\sqrt{10}$ from (0, y, 0). Therefore

$$\sqrt{(1-0)^2 + (2-y)^2 + (3-0)^2} = \sqrt{10}$$

$$\Rightarrow y^2 - 4y + 4 = 0 \Rightarrow (y - 2)^2 = 0 \Rightarrow y = 2$$

Hence, the required point is (0, 2, 0).

Example 10 If a parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes, then find the length of edges of a parallelopiped and length of the diagonal.

Solution Length of edges of the parallelopiped are 5 – 2, 9 – 3, 7 – 5 i.e., 3, 6, 2.

Length of diagonal is $\sqrt{3^2 + 6^2 + 2^2} = 7$ units.

Example 11 Show that the points (0, 7, 10), (–1, 6, 6) and (– 4, 9, 6) form a right angled isosceles triangle.

Solution Let P (0, 7, 10), Q (–1, 6, 6) and R (– 4, 9, 6) be the given three points.

Here $PQ = \sqrt{1+1+16} = 3\sqrt{2}$

$QR = \sqrt{9+9+0} = 3\sqrt{2}$

$PR = \sqrt{16+4+16} = 6$

Now $PQ^2 + QR^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36 = (PR)^2$

Therefore, Δ PQR is a right angled triangle at Q. Also PQ = QR. Hence Δ PQR is an isosceles triangle.

Example 12 Show that the points (5, –1, 1), (7, – 4, 7), (1 – 6, 10) and (–1, – 3, 4) are the vertices of a rhombus.

Solution Let A (5, – 1, 1), B (7, – 4, 7), C(1, – 6, 10) and D (– 1, – 3, 4) be the four points of a quadrilateral. Here

$AB = \sqrt{4+9+36} = 7$, $BC = \sqrt{36+4+9} = 7$, $CD = \sqrt{4+9+36} = 7$,

$DA = \sqrt{23+4+9} = 7$

Note that AB = BC = CD = DA. Therefore, ABCD is a rhombus.

Example 13 Find the ratio in which the line segment joining the points (2, 4, 5) and (3, 5, – 4) is divided by the *xz*-plane.

Solution Let the joint of P (2, 4, 5) and Q (3, 5, – 4) be divided by *xz*-plane in the ratio *k*:1 at the point R(*x*, *y*, *z*). Therefore

$$x = \frac{3k+2}{k+1},\quad y = \frac{5k+4}{k+1},\quad z = \frac{-4k+5}{k+1}$$

Since the point R (*x*, *y*, *z*) lies on the *xz*-plane, the *y*-coordinate should be zero,i.e.,

$$\frac{5k+4}{k+1} = 0 \Rightarrow k = -\frac{4}{5}$$

Hence, the required ratio is – 4 : 5, i.e.; externally in the ratio 4 : 5.

Example 14 Find the coordinate of the point P which is five - sixth of the way from A (– 2, 0, 6) to B (10, – 6, – 12).

Solution Let P (x, y, z) be the required point, i.e., P divides AB in the ratio 5 : 1. Then

$$\text{P}(x, y, z) = \left(\frac{5\times10+1\times-2}{5+1}, \frac{5\times-6+1\times0}{5+1}, \frac{5\times-12+1\times6}{5+1}\right) = (8, -5, -9)$$

Example 15 Describe the vertices and edges of the rectangular parallelopiped with vertex (3, 5, 6) placed in the first octant with one vertex at origin and edges of parallelopiped lie along x, y and z-axes.

Solution The six planes of the parallelopiped are as follows:

Plane OABC lies in the xy-plane. The z-coordinate of every point in this plane is zero. $z = 0$ is the equation of this xy-plane. Plane PDEF is parallel to xy-plane and 6 unit distance above it. The equation of the plane is $z = 6$. Plane ABPF represents plane $x = 3$. Plane OCDE lies in the yz-plane and $x = 0$ is the equation of this plane. Plane AOEF lies in the xz-plane. The y coordinate of everypoint in this plane is zero. Therefore, $y = 0$ is the equation of plane.

Plane BCDP is parallel to the plane AOEF at a distance $y = 5$.

Edge OA lies on the x-axis. The x-axis has equation $y = 0$ and $z = 0$.

Edges OC and OE lie on y-axis and z-axis, respectively. The y-axis has its equation $z = 0, x = 0$. The z-axis has its equation $x = 0$, $y = 0$. The perpendicular distance of the point P (3, 5, 6) from the x-axis is $\sqrt{5^2+6^2} = \sqrt{61}$. The perpendicular distance of the point P (3, 5, 6) from y-axis and z-axis are $\sqrt{3^2+6^2} = \sqrt{45}$ and $\sqrt{3^2+5^2} =$, respectively. The coordinates of the feet of perpendiculars from the point P (3, 5, 6) to the coordinate axes are A, C, E. The coordinates of feet of perpendiculars from the point P on the coordinate planes xy, yz and zx are (3, 5, 0), (0, 5, 6) and (3, 0, 6). Also, perpendicular

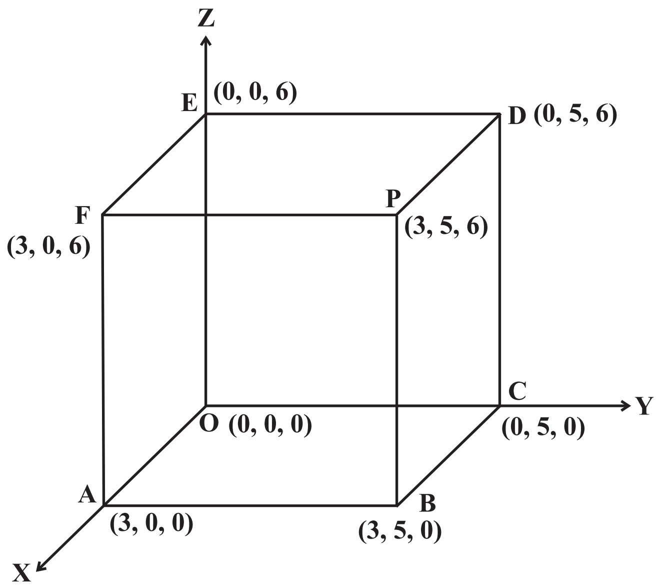

Fig. 12.8

distance of the point P from the *xy*, *yz* and *zx*-planes are 6, 5 and 3, respectively, Fig.12.8.

Example 16 Let A (3, 2, 0), B (5, 3, 2), C (– 9, 6, – 3) be three points forming a triangle. AD, the bisector of $\angle$BAC, meets BC in D. Find the coordinates of the point D.

Solution Note that

$$AB=\sqrt{(5-3)^2+(3-2)^2+(2-0)^2}=\sqrt{4+1+4} = 3$$

$$AC=\sqrt{(-9-3)^2+(6-2)^2+(-3-0)^2}=\sqrt{144+16+9} = 13$$

Since AD is the bisector of $\angle$ BAC,We have $\frac{BD}{DC}=\frac{AB}{AC}=\frac{3}{13}$

i.e., D divides BC in the ratio 3 : 13. Hence, the coordinates of D are

$$\left(\frac{3(-9)+13(5)}{3+13},\frac{3(6)+13(3)}{3+13},\frac{3(-3)+13(2)}{3+13}\right)=\left(\frac{19}{8},\frac{57}{16},\frac{17}{16}\right)$$

Example 17 Determine the point in *yz*-plane which is equidistant from three points A (2, 0 3) B (0, 3, 2) and C (0, 0, 1).

Solution Since *x*-coordinate of every point in *yz*-plane is zero. Let P (0, *y*, *z*) be a point on the *yz*-plane such that PA = PB = PC. Now

PA = PB $\Rightarrow (0-2)^2 + (y-0)^2 + (z-3)^2 = (0-0)^2 + (y-3)^2 + (z-2)^2$, i.e. $z - 3y = 0$

and PB = PC

$\Rightarrow y^2 + 9 - 6y + z^2 + 4 - 4z = y^2 + z^2 + 1 - 2z$, i.e. $3y + z = 6$

Simplifying the two equating, we get $y = 1$, $z = 3$

Here, the coordinate of the point P are (0, 1, 3).

Objective Type Questions

Choose the correct answer out of given four options in each of the Examples from 18 to 23 (M.C.Q.).

Example 18 The length of the foot of perpendicular drawn from the point P (3, 4, 5) on *y*-axis is

(A) 10 (B) $\sqrt{34}$ (C) $\sqrt{113}$ (D) $5\sqrt{2}$

Solution Let *l* be the foot of perpendicular from point P on the *y*-axis. Therefore, its *x* and *z*-coordinates are zero, i.e., (0, 4, 0). Therefore, distance between the points (0, 4, 0) and (3, 4, 5) is $\sqrt{9+25}$ i.e., $\sqrt{34}$.

Example 19 What is the perpendicular distance of the point P (6, 7, 8) from *xy*-plane?

(A) 8 (B) 7 (C) 6 (D) None of these

Solution Let L be the foot of perpendicular drawn from the point P (6, 7, 8) to the *xy*-plane and the distance of this foot L from P is *z*-coordinate of P, i.e., 8 units.

Example 20 L is the foot of the perpendicular drawn from a point P (6, 7, 8) on the *xy*-plane. What are the coordinates of point L?

(A) (6, 0, 0) (B) (6, 7, 0) (C) (6, 0, 8) (D) none of these

Solution Since L is the foot of perpendicular from P on the *xy*-plane, *z*-coordinate is zero in the *xy*-plane. Hence, coordinates of L are (6, 7, 0).

Example 21 L is the foot of the perpendicular drawn from a point (6, 7, 8) on *x*-axis. The coordinates of L are

(A) (6, 0, 0) (B) (0, 7, 0) (C) (0, 0, 8) (D) none of these

Solution Since L is the foot of perpendicular from P on the *x*- axis, *y* and *z*-coordinates are zero. Hence, the coordinates of L are (6, 0, 0).

Example 22 What is the locus of a point for which $y = 0$, $z = 0$?

(A) equation of *x*-axis (B) equation of *y*-axis

(C) equation of *z*-axis (D) none of these

Solution Locus of the point $y = 0, z = 0$ is *x*-axis, since on *x*-axis both $y = 0$ and $z = 0$.

Example 23 L, is the foot of the perpendicular drawn from a point P (3, 4, 5) on the *xz* plane. What are the coordinates of point L ?

(A) (3, 0, 0) (B) (0, 4, 5) (C) (3, 0, 5) (D) (3, 4, 0)

Solution Since L is the foot of perpendicular segment drawn from the point P (3, 4, 5) on the *xz*-plane. Since the *y*-coordinates of all points in the *xz*-plane are zero, coordinate of the foot of perpendicular are (3, 0, 5).

Fill in the blanks in Examples 24 to 28.

Example 24 A line is parallel to *xy*-plane if all the points on the line have equal ______.

Solution A line parallel to *xy*-plane if all the points on the line have equal *z*-coordinates.

Example 25 The equation $x = b$ represents a plane parallel to ______ plane.

Solution Since $x = 0$ represent *yz*-plane, therefore $x = b$ represent a plane parallel to *yz* -plane at a unit distance b from the origin.

Example 26 Perpendicular distance of the point P (3, 5, 6) from *y*-axis is ________

Solution Since M is the foot of perpendicular from P on the *y*-axis, therefore, its *x* and *z*-coordinates are zero. The coordinates of M is (0, 5, 0). Therefore, the perpendicular distance of the point P from *y*-axis $\sqrt{3^2+6^2} = \sqrt{45}$.

Example 27 L is the foot of perpendicular drawn from the point P (3, 4, 5) on *zx*-planes. The coordinates of L are _________.

Solution Since L is the foot of perpendicular from P on the *zx*-plane, *y*-coordinate of every point is zero in the *zx*-plane. Hence, coordinate of L are (3, 0, 5).

Example 28 The length of the foot of perpendicular drawn from the point P (*a*, *b*, *c*) on *z*-axis is ______.

Solution The coordinates of the foot of perpendicular from the point P (*a*, *b*, *c*) on *z*-axis is (0, 0,c). The distance between the point P (*a*, *b*, *c*) and (0, 0, *c*) is $\sqrt{a^2+b^2}$.

Check whether the statements in Example from 30 to 37 are True or False

Example 29 The *y*-axis and *z*-axis, together determine a plane known as *yz*-plane.

Solution True

Example 30 The point (4, 5, – 6) lies in the VIth octant.

Solution False, the point (4, 5, – 6) lies in the Vth octant,

Example 31 The *x*-axis is the intersection of two planes *xy*-plane and *xz* plane.

Solution True.

Example 32 Three mutually perpendicular planes divide the space into 8 octants.

Solution True.

Example 33 The equation of the plane *z* = 6 represent a plane parallel to the *xy*-plane, having a *z*-intercept of 6 units.

Solution True.

Example 34 The equation of the plane *x* = 0 represent the *yz*-plane.

Solution True.

Example 35 The point on the *x*-axis with *x*-coordinate equal to x_0 is written as (x_0, 0, 0).

Solution True.

Example 36 $x = x_0$ represent a plane parallel to the *yz*-plane.

Solution True.

Match each item given under the column C_1 to its correct answer given under column C_2.

Example 37

	Column C_1		**Column C_2**
(a)	If the centriod of the triangle is origin and two of its vertices are (3, – 5, 7) and (–1, 7, – 6) then the third vertex is	(i)	Parallelogram
(b)	If the mid-points of the sides of triangle are (1, 2, – 3), (3, 0, 1) and (–1, 1, – 4) then the centriod is	(ii)	(–2, –2, –1)
(c)	The points (3, – 1, – 1), (5, – 4, 0), (2, 3, – 2) and (0, 6, – 3) are the vertices of a	(iii)	as Isosceles right-angled triangle
(d)	Point A(1, –1, 3), B (2, – 4, 5) and C (5, – 13, 11) are	(iv)	(1, 1, – 2)
(e)	Points A (2, 4, 3), B (4, 1, 9) and C (10, – 1, 6) are the vertices of	(v)	Collinear

Solution (a)

Let A (3, – 5, 7), B (– 1, 7, – 6), C (x, y, z) be the vertices of a Δ ABC with centriod (0, 0, 0)

Therefore, $(0, 0, 0) = \left(\frac{3-1+x}{3}, \frac{-5+7+y}{3}, \frac{7-6+z}{3}\right)$. This implies $\frac{x+2}{3}=0$, $\frac{y+2}{3}=0$, $\frac{z+1}{3}=0$.

Hence x = – 2, y = – 2, and z = – 1.Therefore (a) ↔ (ii)

(b) Let ABC be the given Δ and DEF be the mid-points of the sides BC, CA, AB, respectively. We know that the centriod of the Δ ABC = centriod of Δ DEF.

Therefore, centriod of Δ DEF is $\left(\frac{1+3-1}{3}, \frac{2+0+1}{3}, \frac{-3+1-4}{3}\right)$ = (1, 1, – 2)

Hence (b) $\leftrightarrow$ (iv)

(c) Mid-point of diagonal AC is $\left(\frac{3+2}{2}, \frac{-1+3}{2}, \frac{-1-2}{2}\right) = \left(\frac{5}{2}, 1, \frac{-3}{2}\right)$

Mid-point of diagonal BD is $\left(\frac{5+0}{2}, \frac{-4+6}{2}, \frac{0-3}{2}\right) = \left(\frac{5}{2}, 1, \frac{-3}{2}\right)$

Diagonals of parallelogram bisect each other. Therefore (c) $\leftrightarrow$ (i)

(d) $|AB| = \sqrt{(2-1)^2 + (-4+1)^2 + (5-3)^2} = \sqrt{14}$

$|BC| = \sqrt{(5-2)^2 + (-13+4)^2 + (11-5)^2} = 3\sqrt{14}$

$|AC| = \sqrt{(5-1)^2 + (-13+1)^2 + (11-3)^2} = 4\sqrt{14}$

Now $|AB| + |BC| = |AC|$. Hence Points A, B, C are collinear. Hence (d) $\leftrightarrow$ (v)

(e) AB = $\sqrt{4+9+36} = 7$

BC = $\sqrt{36+4+9} = 7$

CA = $\sqrt{64+25+9} = 7\sqrt{2}$

Now $AB^2 + BC^2 = AC^2$. Hence ABC is an isosceles right angled triangle and hence (e) $\leftrightarrow$ (iii)

12.3 EXERCISE

Short Answer Type

1. Locate the following points:

   (i) (1, – 1, 3), (ii) (– 1, 2, 4)

   (iii) (– 2, – 4, –7) (iv) (– 4, 2, – 5).

2. Name the octant in which each of the following points lies.

   (i) (1, 2, 3), (ii) (4, – 2, 3), (iii) (4, –2, –5) (iv) (4, 2, –5)

   (v) (– 4, 2, 5) (vi) (–3, –1, 6) (vii) (2, – 4, – 7) (viii) (– 4, 2, – 5).

3. Let A, B, C be the feet of perpendiculars from a point P on the *x*, *y*, *z*-axis respectively. Find the coordinates of A, B and C in each of the following where the point P is :

(i) A = (3, 4, 2) (ii) (–5, 3, 7) (iii) (4, – 3, – 5)

4. Let A, B, C be the feet of perpendiculars from a point P on the *xy*, *yz* and *zx*-planes respectively. Find the coordinates of A, B, C in each of the following where the point P is

(i) (3, 4, 5) (ii) (–5, 3, 7) (iii) (4, – 3, – 5).

5. How far apart are the points (2, 0, 0) and (–3, 0, 0)?

6. Find the distance from the origin to (6, 6, 7).

7. Show that if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1-x^2-y^2})$ is at a distance 1 unit from the origin.

8. Show that the point A (1, – 1, 3), B (2, – 4, 5) and (5, – 13, 11) are collinear.

9. Three consecutive vertices of a parallelogram ABCD are A (6, – 2, 4), B (2, 4, – 8), C (–2, 2, 4). Find the coordinates of the fourth vertex.

[**Hint:** Diagonals of a parallelogram have the same mid-point.]

10. Show that the triangle ABC with vertices A (0, 4, 1), B (2, 3, – 1) and C (4, 5, 0) is right angled.

11. Find the third vertex of triangle whose centroid is origin and two vertices are (2, 4, 6) and (0, –2, –5).

12. Find the centroid of a triangle, the mid-point of whose sides are D (1, 2, – 3), E (3, 0, 1) and F (– 1, 1, – 4).

13. The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1). Find its vertices.

14. Three vertices of a Parallelogram ABCD are A (1, 2, 3), B (– 1, – 2, – 1) and C (2, 3, 2). Find the fourth vertex D.

15. Find the coordinate of the points which trisect the line segment joining the points A (2, 1, – 3) and B (5, – 8, 3).

16. If the origin is the centriod of a triangle ABC having vertices A (*a*, 1, 3), B (– 2, *b*, – 5) and C (4, 7, *c*), find the values of *a*, *b*, *c*.

17. Let A (2, 2, – 3), B (5, 6, 9) and C (2, 7, 9) be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D. Find the coordinates of D.

Long Answer Type

18. Show that the three points A (2, 3, 4), B (–1, 2, – 3) and C (– 4, 1, – 10) are collinear and find the ratio in which C divides AB.

19. The mid-point of the sides of a triangle are (1, 5, – 1), (0, 4, – 2) and (2, 3, 4). Find its vertices. Also find the centriod of the triangle.

20. Prove that the points (0, – 1, – 7), (2, 1, – 9) and (6, 5, – 13) are collinear. Find the ratio in which the first point divides the join of the other two.

21. What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?

Objective Type Questions

Choose the correct answer from the given four options inidcated against each of the Exercises from 22 (M.C.Q.).

22. The distance of point P(3, 4, 5) from the *yz*-plane is

(A) 3 units (B) 4 units (C) 5 units (D) 550

23. What is the length of foot of perpendicular drawn from the point P (3, 4, 5) on *y*-axis

(A) $\sqrt{41}$ (B) $\sqrt{34}$ (C) 5 (D) none of these

24. Distance of the point (3, 4, 5) from the origin (0, 0, 0) is

(A) $\sqrt{50}$ (B) 3 (C) 4 (D) 5

25. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of a is

(A) 5 (B) ± 5 (C) – 5 (D) none of these

26. *x*-axis is the intersection of two planes

(A) *xy* and *xz* (B) *yz* and *zx* (C) *xy* and *yz* (D) none of these

27. Equation of *y*-axis is considered as

(A) $x = 0, y = 0$ (B) $y = 0, z = 0$ (C) $z = 0, x = 0$ (D) none of these

28. The point (–2, –3, –4) lies in the

(A) First octant (B) Seventh octant

(C) Second octant (D) Eighth octant

29. A plane is parallel to *yz*-plane so it is perpendicular to :

(A) *x*-axis (B) *y*-axis (C) *z*-axis (D) none of these

30. The locus of a point for which $y = 0, z = 0$ is

(A) equation of *x*-axis (B) equation of *y*-axis

(C) equation at *z*-axis (D) none of these

31. The locus of a point for which $x = 0$ is

(A) *xy*-plane (B) *yz*-plane (C) *zx*-plane (D) none of these

32. If a parallelopiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then the length of diagonal of the parallelopiped is

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

33. L is the foot of the perpendicular drawn from a point P (3, 4, 5) on the *xy*-plane. The coordinates of point L are

(A) (3, 0, 0) (B) (0, 4, 5) (C) (3, 0, 5) (D) none of these

34. L is the foot of the perpendicular drawn from a point (3, 4, 5) on *x*-axis. The coordinates of L are

(A) (3, 0, 0) (B) (0, 4, 0) (C) (0, 0, 5) (D) none of these

Fill in the blanks in Exercises from 35 to 49.

35. The three axes OX, OY, OZ determine ________ .

36. The three planes determine a rectangular parallelopiped which has ________ of rectangular faces.

37. The coordinates of a point are the perpendicular distance from the ________ on the respectives axes.

38. The three coordinate planes divide the space into ________ parts.

39. If a point P lies in *yz*-plane, then the coordinates of a point on *yz*-plane is of the form ________.

40. The equation of *yz*-plane is ________.

41. If the point P lies on *z*-axis, then coordinates of P are of the form ________.

42. The equation of *z*-axis, are ________.

43. A line is parallel to *xy*-plane if all the points on the line have equal ________.

44. A line is parallel to *x*-axis if all the points on the line have equal ________.

45. $x = a$ represent a plane parallel to ________.

46. The plane parallel to *yz* - plane is perpendicular to ________.

47. The length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 10, 13 and 8 units are ______.

48. If the distance between the points $(a, 2, 1)$ and $(1, -1, 1)$ is 5, then a ________.

49. If the mid-points of the sides of a triangle AB; BC; CA are D $(1, 2, -3)$, E $(3, 0, 1)$ and F $(-1, 1, -4)$, then the centriod of the triangle ABC is ________.

50. Match each item given under the column C_1 to its correct answer given under column C_2.

	Column C_1		**Column C_2**
(b)	Point (2, 3,4) lies in the	(ii)	yz-plane
(c)	Locus of the points having x coordinate 0 is	(iii)	z-coordinate is zero
(d)	A line is parallel to x-axis if and only	(iv)	z-axis
(e)	If $x = 0$, $y = 0$ taken together will represent the	(v)	plane parallel to xy-plane
(f)	$z = c$ represent the plane	(vi)	if all the points on the line have equal y and z-coordinates.
(g)	Planes $x = a$, $y = b$ represent the line	(vii)	from the point on the respective
(h)	Coordinates of a point are the distances from the origin to the feet of perpendiculars	(viii)	parallel to z - axis.
(i)	A ball is the solid region in the space enclosed by a	(ix)	disc
(j)	Region in the plane enclosed by a circle is known as a	(x)	sphere