Chapter 1

SETS

1.1 Overview

This chapter deals with the concept of a set, operations on sets.Concept of sets will be useful in studying the relations and functions.

1.1.1 *Set and their representations* A set is a well-defined collection of objects. There are two methods of representing a set

(i) Roaster or tabular form (ii) Set builder form

1.1.2 *The empty set* A set which does not contain any element is called the empty set or the void set or null set and is denoted by { } or ϕ.

1.1.3 *Finite and infinite sets* A set which consists of a finite number of elements is called a finite set otherwise, the set is called an infinite set.

1.1.4 *Subsets* A set A is said to be a subset of set B if every element of A is also an element of B. In symbols we write $A \subset B$ if $a \in A \Rightarrow a \in B$.

We denote set of real numbers by **R**

set of natural numbers by **N**

set of integers by **Z**

set of rational numbers by **Q**

set of irrational numbers by **T**

We observe that

$$\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R},$$

$$\mathbf{T} \subset \mathbf{R},\ \mathbf{Q} \not\subset \mathbf{T},\ \mathbf{N} \not\subset \mathbf{T}$$

1.1.5 *Equal sets* Given two sets A and B, if every elements of A is also an element of B and if every element of B is also an element of A, then the sets A and B are said to be equal. The two equal sets will have exactly the same elements.

1.1.6 *Intervals as subsets of R* Let $a, b \in$ R and $a < b$. Then

(a) An open interval denoted by (a, b) is the set of real numbers $\{x : a < x < b\}$

(b) A closed interval denoted by $[a, b]$ is the set of real numbers $\{x : a \leq x \leq b)$

(c) Intervals closed at one end and open at the other are given by

$[a, b) = \{x : a \leq x < b\}$

$(a, b] = \{x : a < x \leq b\}$

1.1.7 *Power set* The collection of all subsets of a set A is called the **power set** of A. It is denoted by P(A). If the number of elements in A = n , i.e., $n(\text{A}) = n$, then the number of elements in P(A) = 2^n.

1.1.8 *Universal set* This is a basic set; in a particular context whose elements and subsets are relevant to that particular context. For example, for the set of vowels in English alphabet, the universal set can be the set of all alphabets in English. Universal set is denoted by **U**.

1.1.9 *Venn diagrams* Venn Diagrams are the diagrams which represent the relationship between sets. For example, the set of natural numbers is a subset of set of whole numbers which is a subset of integers. We can represent this relationship through Venn diagram in the following way.

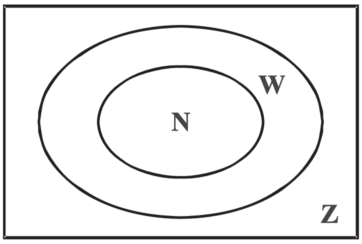

Fig 1.1

1.1.10 *Operations on sets*

Union of Sets : The union of any two given sets A and B is the set C which consists of all those elements which are either in A or in B. In symbols, we write

$$C = A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

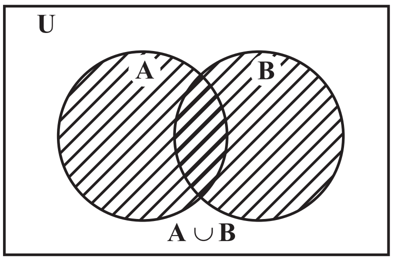

Fig 1.2 (a)

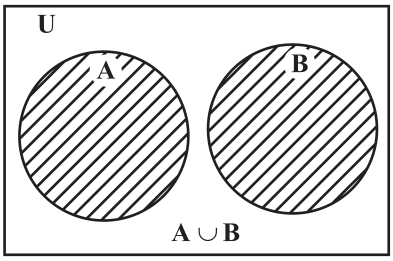

Fig 1.2 (b)

Some properties of the operation of union.

(i) $A \cup B = B \cup A$

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$

(iii) $A \cup \phi = A$

(iv) $A \cup A = A$

(v) $U \cup A = U$

Intersection of sets: The intersection of two sets A and B is the set which consists of all those elements which belong to both A and B. Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

When $A \cap B = \phi$, then A and B are called disjoint sets.

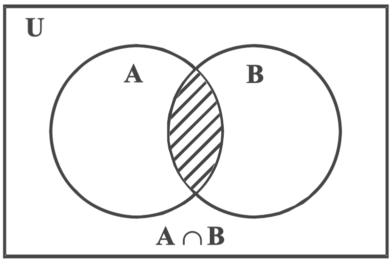

Fig 1.3 (a)

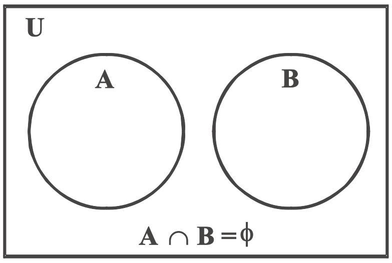

Fig 1.3 (b)

Some properties of the operation of intersection

(i) $A \cap B = B \cap A$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) $\phi \cap A = \phi$; $U \cap A = A$ (iv) $A \cap A = A$

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(vi) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Difference of sets The difference of two sets A and B, denoted by A – B is defined as set of elements which belong to A but not to B. We write

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

also,
$$B - A = \{ x : x \in B \text{ and } x \notin A\}$$

Complement of a set Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write

$$A' = \{x : x \in U \text{ and } x \notin A\}. \text{ Also } A' = U - A$$

Some properties of complement of sets

(i) Law of complements:

(a) $A \cup A' = U$ (b) $A \cap A' = \phi$

(ii) De Morgan's law

(a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$

(iii) $(A')' = A$

(iv) $U' = \phi$ and $\phi' = U$

1.1.11 *Formulae to solve practical problems on union and intersection of two sets*

Let A, B and C be any finite sets. Then

(a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(b) If $(A \cap B) = \phi$, then $n(A \cup B) = n(A) + n(B)$

(c) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

1.2 Solved Examples

Short Answer Type

Example 1 Write the following sets in the roaster form.

(i) $A = \{x \mid x$ is a positive integer less than 10 and $2^x - 1$ is an odd number$\}$

(ii) $C = \{x : x^2 + 7x - 8 = 0, x \in \mathbf{R}\}$

Solution

(i) $2^x - 1$ is always an odd number for all positive integral values of x. In particular, $2^x - 1$ is an odd number for $x = 1, 2, \ldots, 9$. Thus, $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(ii) $x^2 + 7x - 8 = 0$ or $(x + 8)(x - 1) = 0$ giving $x = -8$ or $x = 1$

Thus, $C = \{-8, 1\}$

Example 2 State which of the following statements are true and which are false. Justify your answer.

(i) $37 \notin \{x \mid x$ has exactly two positive factors$\}$

(ii) $28 \in \{y \mid$ the sum of the all positive factors of y is $2y\}$

(iii) $7{,}747 \in \{t \mid t$ is a multiple of $37\}$

Solution

(i) False

Since, 37 has exactly two positive factors, 1 and 37, 37 belongs to the set.

(ii) True

Since, the sum of positive factors of 28

$$= 1 + 2 + 4 + 7 + 14 + 28$$
$$= 56 = 2(28)$$

(iii) False

7,747 is not a multiple of 37.

Example 3 If X and Y are subsets of the universal set U, then show that

(i) $Y \subset X \cup Y$ (ii) $X \cap Y \subset X$ (iii) $X \subset Y \Rightarrow X \cap Y = X$

Solution

(i) $X \cup Y = \{x \mid x \in X$ or $x \in Y\}$

Thus $x \in Y \Rightarrow x \in X \cup Y$

Hence, $Y \subset X \cup Y$

(ii) $X \cap Y = \{x \mid x \in X \text{ and } x \in Y\}$

Thus $x \in X \cap Y \Rightarrow x \in X$

Hence $X \cap Y \subset X$

(iii) Note that

$x \in X \cap Y \Rightarrow x \in X$

Thus $X \cap Y \subset X$

Also, since $X \subset Y$,

$x \in X \Rightarrow x \in Y \Rightarrow x \in X \cap Y$

so that $X \subset X \cap Y$

Hence the result $X = X \cap Y$ follows.

Example 4 Given that N = {1, 2, 3, ..., 100}, then

(i) Write the subset A of N, whose element are odd numbers.

(ii) Write the subset B of N, whose element are represented by $x + 2$, where $x \in N$.

Solution

(i) $A = \{x \mid x \in N \text{ and } x \text{ is odd}\} = \{1, 3, 5, 7, ..., 99\}$

(ii) $B = \{y \mid y = x + 2, x \in N\}$

So, for $1 \in N$, $y = 1 + 2 = 3$

$2 \in N$, $y = 2 + 2 = 4$,

and so on. Therefore, B = {3, 4, 5, 6, ... , 100}

Example 5 Given that E = {2, 4, 6, 8, 10}. If n represents any member of E, then, write the following sets containing all numbers represented by

(i) $n + 1$ (ii) n^2

Solution Given E = {2, 4, 6, 8, 10}

(i) Let $A = \{x \mid x = n + 1, n \in E\}$

Thus, for $2 \in E$, $x = 3$

$4 \in E$, $x = 5$,

and so on. Therefore, A = {3, 5, 7, 9, 11}.

(ii) Let $B = \{x \mid x = n^2, n \in E\}$

So, for $2 \in E$, $x = (2)^2 = 4$, $4 \in E$, $x = (4)^2 = 16$, $6 \in E$, $x = (6)^2 = 36$,

and so on. Hence, B = {4, 16, 36, 64, 100}

Example 6 Let X = {1, 2, 3, 4, 5, 6}. If n represent any member of X, express the following as sets:

(i) $n \in$ X but $2n \notin$ X (ii) $n + 5 = 8$

(iii) n is greater than 4.

Solution

(i) For X = {1, 2, 3, 4, 5, 6}, it is the given that $n \in$ X, but $2n \notin$ X.

Let, A = $\{x \mid x \in$ X and $2x \notin$ X$\}$

Now, $1 \notin$ A as $2.1 = 2 \in$ X

$2 \notin$ A as $2.2 = 4 \in$ X

$3 \notin$ A as $2.3 = 6 \in$ X

But $4 \in$ A as $2.4 = 8 \notin$ X

$5 \in$ A as $2.5 = 10 \notin$ X

$6 \in$ A as $2.6 = 12 \notin$ X

So, A = {4, 5, 6}

(ii) Let B = $\{x \mid x \in$ X and $x + 5 = 8\}$

Here, B = {3}

as $x = 3 \in$ X and $3 + 5 = 8$ and there is no other element belonging to X such that $x + 5 = 8$.

(iii) Let C = $\{x \mid x \in$ X, $x > 4\}$

Therefore, C = {5, 6}

Example 7 Draw the Venn diagrams to illustrate the followoing relationship among sets E, M and U, where E is the set of students studying English in a school, M is the set of students studying Mathematics in the same school, U is the set of all students in that school.

(i) All the students who study Mathematics study English, but some students who study English do not study Mathematics.

(ii) There is no student who studies both Mathematics and English.

(iii) Some of the students study Mathematics but do not study English, some study English but do not study Mathematics, and some study both.

(iv) Not all students study Mathematics, but every students studying English studies Mathematics.

Solution

(i) Since all of the students who study mathematics study English, but some students who study English do not study Mathematics.

Therefore, $M \subset E \subset U$

Thus the Venn Diagram is

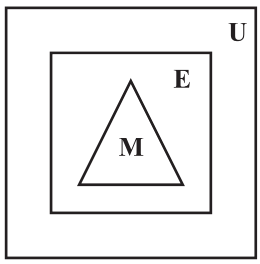

Fig 1.4

(ii) Since there is no student who study both English and Mathematics

Hence, $E \cap M = \phi$.

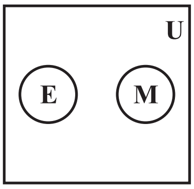

Fig 1.5

(iii) Since there are some students who study both English and Mathematics, some English only and some Mathematics only.

Thus, the Venn Diagram is

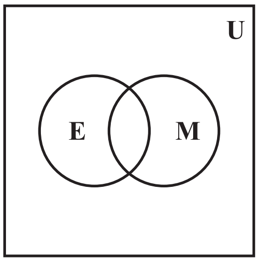

Fig 1.6

(iv) Since every student studying English studiesMathematics.

Hence, $E \subset M \subset U$

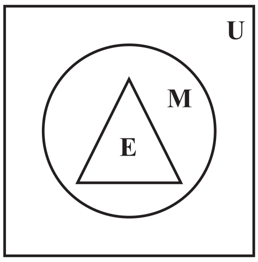

Fig 1.7

Example 8 For all sets A, B and C

Is $(A \cap B) \cup C = A \cap (B \cup C)$?

Justify your statement.

Solution No. consider the following sets A, B and C :

$$A = \{1, 2, 3\}$$
$$B = \{2, 3, 5\}$$
$$C = \{4, 5, 6\}$$

Now $(A \cap B) \cup C = (\{1, 2, 3\} \cap \{2, 3, 5\}) \cup \{4, 5, 6\}$

$= \{2, 3\} \cup \{4, 5, 6\}$

$= \{2, 3, 4, 5, 6\}$

And $A \cap (B \cup C) = \{1, 2, 3\} \cap [\{2, 3, 5\} \cup \{4, 5, 6\}$

$= \{1, 2, 3\} \cap \{2, 3, 4, 5, 6\}$

$= \{2, 3\}$

Therefore, $(A \cap B) \cup C \neq A \cap (B \cup C)$

Example 9 Use the properties of sets to prove that for all the sets A and B

$$A - (A \cap B) = A - B$$

Solution We have

$A - (A \cap B) = A \cap (A \cap B)'$ (since $A - B = A \cap B'$)

$= A \cap (A' \cup B')$ [by De Morgan's law)

$= (A \cap A') \cup (A \cap B')$ [by distributive law]

$= \phi \cup (A \cap B')$

$= A \cap B' = A - B$

Long Answer Type

Example 10 For all sets A, B and C

Is $(A - B) \cap (C - B) = (A \cap C) - B$?

Justify your answer.

Solution Yes

Let $x \in (A - B) \cap (C - B)$

$\Rightarrow$ $x \in A - B$ and $x \in C - B$

$\Rightarrow$ $(x \in A$ and $x \notin B)$ and $(x \in C$ and $x \notin B)$

$\Rightarrow$ $(x \in A$ and $x \in C)$ and $x \notin B$

$\Rightarrow$ $(x \in A \cap C)$ and $x \notin B$

$\Rightarrow$ $x \in (A \cap C) - B$

So $(A - B) \cap (C - B) \subset (A \cap C) - B$ ... (1)

Now, conversely

Let $\quad y \in (A \cap C) - B$

$\Rightarrow \quad y \in (A \cap C)$ and $y \notin B$

$\Rightarrow \quad (y \in A$ and $y \in C)$ and $(y \notin B)$

$\Rightarrow \quad (y \in A$ and $y \notin B)$ and $(y \in C$ and $y \notin B)$

$\Rightarrow \quad y \in (A - B)$ and $y \in (C - B)$

$\Rightarrow \quad y \in (A - B) \cap (C - B)$

So $\quad (A \cap C) - B \subset (A - B) \cap (C - B) \quad \ldots (2)$

From (1) and (2), $(A - B) \cap (C - B) = (A \cap C) - B$

Example 11 Let A, B and C be sets. Then show that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Solution We first show that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$. Then

$\quad x \in A \quad$ or $\quad x \in B \cap C$

$\Rightarrow \quad x \in A \quad$ or $\quad (x \in B$ and $x \in C)$

$\Rightarrow \quad (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow \quad (x \in A \cup B)$ and $\quad (x \in A \cup C)$

$\Rightarrow \quad x \in (A \cup B) \cap (A \cup C)$

Thus, $\quad A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C) \quad \ldots (1)$

Now we will show that $(A \cup B) \cap (A \cup C) \subset (A \cup C)$

Let $\quad x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow \quad x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow \quad (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow \quad x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow \quad x \in A$ or $(x \in B \cap C)$

$\Rightarrow \quad x \in A \cup (B \cap C)$

Thus, $\quad (A \cup B) \cap (A \cup C) \subset A \cup (B \cap C) \quad \ldots (2)$

So, from (1) and (2), we have

$$A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$$

Example 12 Let P be the set of prime numbers and let $S = \{t \mid 2^t - 1$ is a prime$\}$. Prove that $S \subset P$.

Solution Now the equivalent contrapositive statement of $x \in S \Rightarrow x \in P$ is $x \notin P \Rightarrow x \notin S$.

Now, we will prove the above contrapositive statement by contradiction method

Let $x \notin \mathrm{P}$

$\Rightarrow$ x is a composite number

Let us now assume that $x \in \mathrm{S}$

$\Rightarrow$ $2^x - 1 = m$ (where m is a prime number)

$\Rightarrow$ $2^x = m + 1$

Which is not true for all composite number, say for $x = 4$ because

$2^4 = 16$ which can not be equal to the sum of any prime number m and 1.

Thus, we arrive at a contradiction

$\Rightarrow$ $x \notin \mathrm{S}$.

Thus, when $x \notin \mathrm{P}$, we arrive at $x \notin \mathrm{S}$

So $\mathrm{S} \subset \mathrm{P}$.

Example 13 From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry. What is the largest possible number that could have passed all three examination?

Solution Let M be the set of students passing in Mathematics

P be the set of students passing in Physics

C be the set of students passing in Chemistry

Now, $n(\mathrm{M} \cup \mathrm{P} \cup \mathrm{C}) = 50$, $n(\mathrm{M}) = 37$, $n(\mathrm{P}) = 24$, $n(\mathrm{C}) = 43$

$n(\mathrm{M} \cap \mathrm{P}) \leq 19$, $n(\mathrm{M} \cap \mathrm{C}) \leq 29$, $n(\mathrm{P} \cap \mathrm{C}) \leq 20$ (Given)

$$n(\mathrm{M} \cup \mathrm{P} \cup \mathrm{C}) = n(\mathrm{M}) + n(\mathrm{P}) + n(\mathrm{C}) - n(\mathrm{M} \cap \mathrm{P}) - n(\mathrm{M} \cap \mathrm{C}) - n(\mathrm{P} \cap \mathrm{C}) + n(\mathrm{M} \cap \mathrm{P} \cap \mathrm{C}) \leq 50$$

$\Rightarrow$ $37 + 24 + 43 - 19 - 29 - 20 + n(\mathrm{M} \cap \mathrm{P} \cap \mathrm{C}) \leq 50$

$\Rightarrow$ $n(\mathrm{M} \cap \mathrm{P} \cap \mathrm{C}) \leq 50 - 36$

$\Rightarrow$ $n(\mathrm{M} \cap \mathrm{P} \cap \mathrm{C}) \leq 14$

Thus, the largest possible number that could have passed all the three examinations is 14.

Objective Type Questions

Choose the correct answer from the given four options in each of the Examples 14 to 16 : (M.C.Q.)

Example 14 Each set X_r contains 5 elements and each set Y_r contains 2 elements and $\bigcup_{r=1}^{20} X_r = S = \bigcup_{r=1}^{n} Y_r$. If each element of S belong to exactly 10 of the X_r's and to exactly 4 of the Y_r's, then n is

(A) 10 (B) 20 (C) 100 (D) 50

Solution The correct answer is (B)

Since, $n(X_r) = 5$, $\bigcup_{r=1}^{20} X_r = S$, we get $n(S) = 100$

But each element of S belong to exactly 10 of the X_r's

So, $\frac{100}{10} = 10$ are the number of distinct elements in S.

Also each element of S belong to exactly 4 of the Y_r's and each Y_r contain 2 elements. If S has n number of Y_r in it. Then

$$\frac{2n}{4} = 10$$

which gives $n = 20$

Example 15 Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are.

(A) 7, 6 (B) 5, 1 (C) 6, 3 (D) 8, 7

Solution The correct answer is (C).

Since, let A and B be such sets, i.e., $n(A) = m$, $n(B) = n$

So $n(P(A)) = 2^m$, $n(P(B)) = 2^n$

Thus $n(P(A)) - n(P(B)) = 56$, i.e., $2^m - 2^n = 56$

$\Rightarrow$ $2^n(2^{m-n} - 1) = 2^3 7$

$\Rightarrow$ $n = 3$, $2^{m-n} - 1 = 7$

$\Rightarrow$ $m = 6$

Example 16 The set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

(A) $B \cap C'$ (B) $A \cap C$ (C) $B \cup C'$ (D) $A \cap C'$

Solution The correct choice is (A).

Since $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$

$= (A \cup (B \cup C)) \cap (A' \cup (B \cup C)) \cap C'$
$= (A \cap A') \cup (B \cup C) \cap C'$
$= \phi \cup (B \cup C) \cap C'$
$= B \cap C' \cup \phi = B \cap C'$

Fill in the blanks in Examples 17 and 18 :

Example 17 If A and B are two finite sets, then $n(A) + n(B)$ is equal to ___________

Solution Since $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

So $n(A) + n(B) = n(A \cup B) + n(A \cap B)$

Example 18 If A is a finite set containing n element, then number of subsets of A is ___________

Solution 2^n

State true or false for the following statements given in Examples 19 and 20.

Example 19 Let R and S be the sets defined as follows:

$R = \{x \in \mathbf{Z} \mid x \text{ is divisible by } 2\}$

$S = \{y \in \mathbf{Z} \mid y \text{ is divisible by } 3\}$

then $R \cap S = \phi$

Solution False

Since 6 is divisible by both 3 and 2.

Thus $R \cap S \neq \phi$

Example 20 $\mathbf{Q} \cap \mathbf{R} = \mathbf{Q}$, where $\mathbf{Q}$ is the set of rational numbers and $\mathbf{R}$ is the set of real numbers.

Solution True

Since $\mathbf{Q} \subset \mathbf{R}$

So $\mathbf{Q} \cap \mathbf{R} = \mathbf{Q}$

1.3 EXERCISE

Short Answer Type

1. Write the following sets in the roaster from

(i) $A = \{x : x \in \mathbf{R}, 2x + 11 = 15\}$ (ii) $B = \{x \mid x^2 = x, x \in \mathbf{R}\}$

(iii) $C = \{x \mid x$ is a positive factor of a prime number $p\}$

2. Write the following sets in the roaster form :

(i) $D = \{t \mid t^3 = t, t \in R\}$ (ii) $E = \{w \mid \frac{w-2}{w+3} = 3, w \in \mathbf{R}\}$

(iii) $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in \mathbf{R}\}$

3. If $Y = \{x \mid x$ is a positive factor of the number $2^{p-1}(2^p - 1)$, where $2^p - 1$ is a prime number$\}$. Write Y in the roaster form.

4. State which of the following statements are true and which are false. Justify your answer.

(i) $35 \in \{x \mid x$ has exactly four positive factors$\}$.

(ii) $128 \in \{y \mid$ the sum of all the positive factors of y is $2y\}$

(iii) $3 \notin \{x \mid x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$

(iv) $496 \notin \{y \mid$ the sum of all the positive factors of y is $2y\}$.

5. Given $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$

Verify that $L - (M \cup N) = (L - M) \cap (L - N)$

6. If A and B are subsets of the universal set U, then show that

(i) $A \subset A \cup B$ (ii) $A \subset B \Leftrightarrow A \cup B = B$

(iii) $(A \cap B) \subset A$

7. Given that $N = \{1, 2, 3, ..., 100\}$. Then write

(i) the subset of N whose elements are even numbers.

(ii) the subset of N whose element are perfect square numbers.

8. If $X = \{1, 2, 3\}$, if n represents any member of X, write the following sets containing all numbers represented by

(i) $4n$ (ii) $n + 6$ (iii) $\frac{n}{2}$ (iv) $n - 1$

9. If $Y = \{1, 2, 3, ... 10\}$, and a represents any element of Y, write the following sets, containing all the elements satisfying the given conditions.

(i) $a \in Y$ but $a^2 \notin Y$ (ii) $a + 1 = 6, a \in Y$

(iii) a is less than 6 and $a \in Y$

10. A, B and C are subsets of Universal Set U. If $A = \{2, 4, 6, 8, 12, 20\}$

$B = \{3, 6, 9, 12, 15\}$, $C = \{5, 10, 15, 20\}$ and U is the set of all whole numbers, draw a Venn diagram showing the relation of U, A, B and C.

11. Let U be the set of all boys and girls in a school, G be the set of all girls in the school, B be the set of all boys in the school, and S be the set of all students in the school who take swimming. Some, but not all, students in the school take swimming. Draw a Venn diagram showing one of the possible interrelationship among sets U, G, B and S.

12. For all sets A, B and C, show that $(A - B) \cap (C - B) = A - (B \cup C)$

Determine whether each of the statement in Exercises 13 – 17 is true or false. Justify your answer.

13. For all sets A and B, $(A - B) \cup (A \cap B) = A$

14. For all sets A, B and C, $A - (B - C) = (A - B) - C$

15. For all sets A, B and C, if $A \subset B$, then $A \cap C \subset B \cap C$

16. For all sets A, B and C, if $A \subset B$, then $A \cup C \subset B \cup C$

17. For all sets A, B and C, if $A \subset C$ and $B \subset C$, then $A \cup B \subset C$.

Using properties of sets prove the statements given in Exercises 18 to 22

18. For all sets A and B, $A \cup (B - A) = A \cup B$

19. For all sets A and B, $A - (A - B) = A \cap B$

20. For all sets A and B, $A - (A \cap B) = A - B$

21. For all sets A and B, $(A \cup B) - B = A - B$

22. Let $T = \left\{ x \mid \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x} \right\}$. Is T an empty set? Justify your answer.

Long Answer Type

23. Let A, B and C be sets. Then show that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

24. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science; 4 in English and Science; 4 in all the three. Find how many passed

(i) in English and Mathematics but not in Science

(ii) in Mathematics and Science but not in English

(iii) in Mathematics only

(iv) in more than one subject only

25. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Find the number of students who play neither?

26. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

27. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find

(a) The number of families which buy newspaper A only.

(b) The number of families which buy none of A, B and C

28. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows:
French = 17, English = 13, Sanskrit = 15
French and English = 09, English and Sanskrit = 4

French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study

(i) French only
(ii) English only
(iii) Sanskrit only
(iv) English and Sanskrit but not French
(v) French and Sanskrit but not English
(vi) French and English but not Sanskrit
(vii) at least one of the three languages
(viii) none of the three languages

Objective Type Questions

Choose the correct answers from the given four options in each Exercises 29 to 43 (M.C.Q.).

29. Suppose $A_1, A_2, ..., A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, ..., B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j'S. then n is equal to

(A) 15 (B) 3 (C) 45 (D) 35

30. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively,

(A) 4, 7 (B) 7, 4 (C) 4, 4 (D) 7, 7

31. The set $(A \cap B')' \cup (B \cap C)$ is equal to

(A) $A' \cup B \cup C$ (B) $A' \cup B$ (C) $A' \cup C'$ (D) $A' \cap B$

32. Let F_1 be the set of parallelograms, F_2 the set of rectangles, F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to

(A) $F_2 \cap F_3$ (B) $F_3 \cap F_4$

(C) $F_2 \cup F_5$ (D) $F_2 \cup F_3 \cup F_4 \cup F_1$

33. Let S = set of points inside the square, T = the set of points inside the triangle and C = the set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then

(A) $S \cap T \cap C = \phi$ (B) $S \cup T \cup C = C$

(C) $S \cup T \cup C = S$ (D) $S \cup T = S \cap C$

34. Let R be set of points inside a rectangle of sides a and b ($a, b > 1$) with two sides along the positive direction of x-axis and y-axis. Then

(A) $R = \{(x, y) : 0 \leq x \leq a, 0 \leq y \leq b\}$

(B) $R = \{(x, y) : 0 \leq x < a, 0 \leq y \leq b\}$

(C) $R = \{(x, y) : 0 \leq x \leq a, 0 < y < b\}$

(D) $R = \{(x, y) : 0 < x < a, 0 < y < b\}$

35. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Then, the number of students who play neither is

(A) 0 (B) 25 (C) 35 (D) 45

36. In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Then the number of persons who read neither is

(A) 210 (B) 290 (C) 180 (D) 260

37. If $X = \{8^n - 7n - 1 \mid n \in \mathbf{N}\}$ and $Y = \{49n - 49 \mid n \in \mathbf{N}\}$. Then

(A) $X \subset Y$ (B) $Y \subset X$ (C) $X = Y$ (D) $X \cap Y = \phi$

38. A survey shows that 63% of the people watch a News Channel whereas 76% watch another channel. If x% of the people watch both channel, then

(A) $x = 35$ (B) $x = 63$ (C) $39 \leq x \leq 63$ (D) $x = 39$

39. If sets A and B are defined as

$A = \{(x, y) \mid y = \frac{1}{x}, 0 \neq x \in \mathbf{R}\}$ $B = \{(x, y) \mid y = -x, x \in \mathbf{R}\}$, then

(A) $A \cap B = A$ (B) $A \cap B = B$ (C) $A \cap B = \phi$ (D) $A \cup B = A$

40. If A and B are two sets, then $A \cap (A \cup B)$ equals

(A) A (B) B (C) ϕ (D) $A \cap B$

41. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$ $B = \{2, 4, ..., 18\}$ and **N** the set of natural numbers is the universal set, then $A' \cup ((A \cup B) \cap B')$ is

(A) ϕ (B) **N** (C) A (D) B

42. Let $S = \{x \mid x$ is a positive multiple of 3 less than 100$\}$

$P = \{x \mid x$ is a prime number less than 20$\}$. Then $n(S) + n(P)$ is

(A) 34 (B) 31 (C) 33 (D) 30

43. If X and Y are two sets and X' denotes the complement of X, then $X \cap (X \cup Y)'$ is equal to

(A) X (B) Y (C) ϕ (D) $X \cap Y$

Fill in the blanks in each of the Exercises from 44 to 51 :

44. The set $\{x \in \mathbf{R} : 1 \le x < 2\}$ can be written as _______________.

45. When $A = \phi$, then number of elements in P(A) is _______________.

46. If A and B are finite sets such that $A \subset B$, then $n(A \cup B) =$ _______________.

47. If A and B are any two sets, then $A - B$ is equal to _______________.

48. Power set of the set $A = \{1, 2\}$ is _______________.

49. Given the sets $A = \{1, 3, 5\}$. $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$. Then the universal set of all the three sets A, B and C can be _______________.

50. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 5\}$, $B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$. Then

(i) $(B \cup C)'$ is _______________. (ii) $(C - A)'$ is _______________.

51. For all sets A and B, $A - (A \cap B)$ is equal to _______________.

52. Match the following sets for all sets A, B and C

(i)	$((A' \cup B') - A)'$	(a)	$A - B$
(ii)	$[B' \cup (B' - A)]'$	(b)	A
(iii)	$(A - B) - (B - C)$	(c)	B
(iv)	$(A - B) \cap (C - B)$	(d)	$(A \times B) \cap (A \times C)$
(v)	$A \times (B \cap C)$	(e)	$(A \times B) \cup (A \times C)$
(vi)	$A \times (B \cup C)$	(f)	$(A \cap C) - B$

State True or False for the following statements in each of the Exercises from 53 to 58 :

53. If A is any set, then $A \subset A$

54. Given that $M = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and if $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, then $B \not\subset M$

55. The sets $\{1, 2, 3, 4\}$ and $\{3, 4, 5, 6\}$ are equal.

56. $\mathbf{Q} \cup \mathbf{Z} = \mathbf{Q}$, where $\mathbf{Q}$ is the set of rational numbers and $\mathbf{Z}$ is the set of integers.

57. Let sets R and T be defined as

$R = \{x \in \mathbf{Z} \mid x \text{ is divisible by } 2\}$

$T = \{x \in \mathbf{Z} \mid x \text{ is divisible by } 6\}$. Then $T \subset R$

58. Given $A = \{0, 1, 2\}$, $B = \{x \in \mathbf{R} \mid 0 \le x \le 2\}$. Then $A = B$.